Nonconservative Dynamics: The formation of Chimera States and Stability Through Asymmetry in Nonlinear Optics

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"Imperfect Chimeras in a Ring of 4 - dimensional Simplified Lorenz Systems"

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Outline of the Lecture

1. The Li – Sprott Oscillator: A Simple 4-D Lorenz like System
2. Coexistence of Limit Cycles and Strange Attractors
3. The formation of "perfect" and imperfect" chimeras
4. A simple analytical calculation of a limit cycle
5. Stable Steady States of Asymmetric Active Couplers
6. Stable Steady States of Semiconductor Lasers
7. Hopf Bifurcations and…. Chimera States in Laser Arrays?

1. The Li-Sprott Oscillator: A Simple 4-D Lorenz – like System

Let us consider the 4-D Li-Sprott nonlinear Oscillator:

$$\dot{x} = y - \mathbf{x}$$

$$\dot{y} = -\mathbf{xz} + \mathbf{u}$$

$$\dot{z} = xy - \mathbf{a} \qquad (1)$$

$$\dot{u} = -by$$

first studied in *Li C, Sprott JC, "Coexisting hidden attractors in a 4-D simplified Lorenz system", Int. J. Bifurc. Chaos, 2014, 24 1450034,* where **a, b** are fixed parameters.

This system has no fixed points and can remarkably support *the coexistence of stable limit cycles and strange attractors!*

2. Coexistence of limit cycle and strange attractors

Fig. 1. Limit cycle coexists with a symmetric pair of strange attractors at a = 7, b = 0.1 (green and blue attractors correspond to symmetric initial conditions and red limit cycle occurs near the origin.

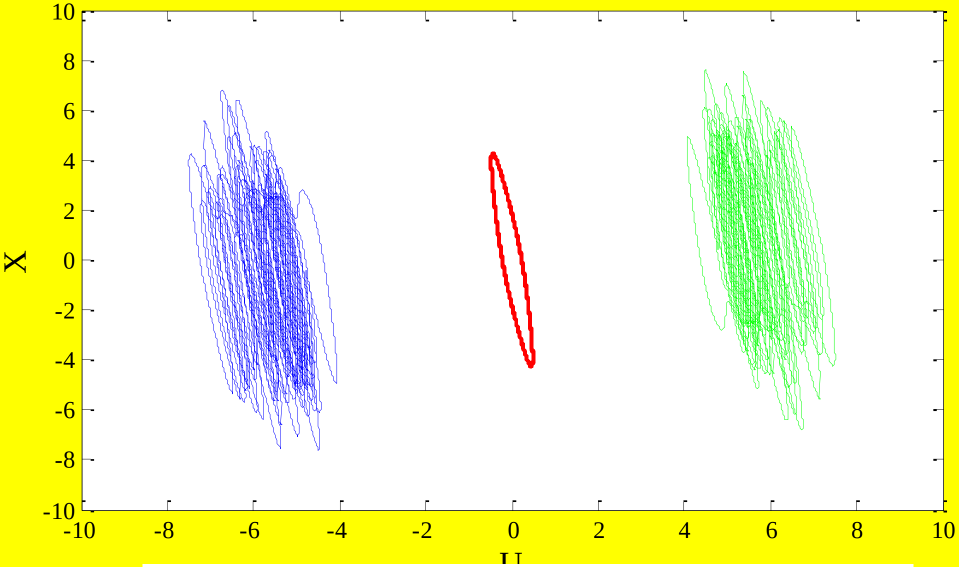
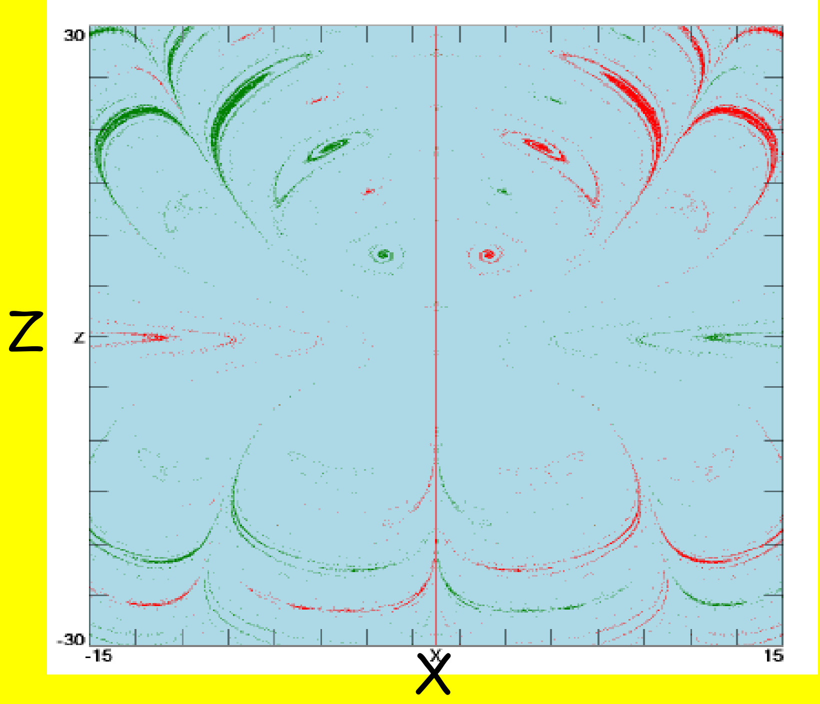
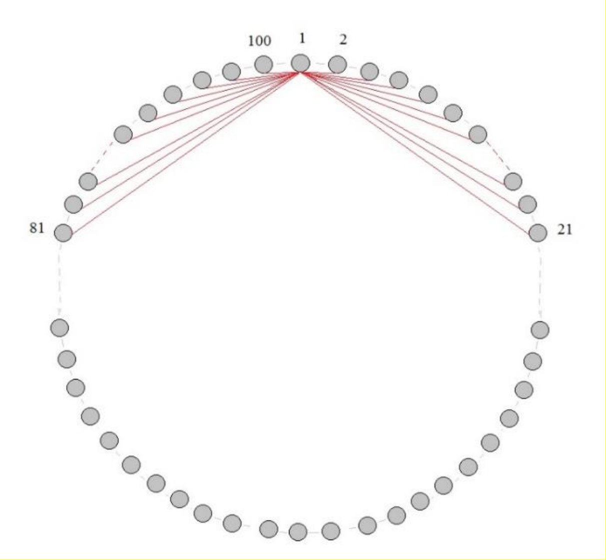


Fig. 2. What is particularly interesting are the intertwined basins of attraction of these solutions shown on the right (blue for the limit cycle, green and red for the strange attractors.



Let us now consider 100 of these oscillators on a ring of the form:



where each oscillator is coupled to P = 20 of its right and 20 of its left neighbors, obeying the equations of motion:

$$\dot{x}_i = y_i - x_i + \frac{d}{2P}\sum_{j=(i-P)\bmod N}^{(i+P)\bmod N}[x_j - x_i]$$

$$\dot{y}_i = -x_i z_i + u_i$$

$$\dot{z}_i = x_i y_i - a$$

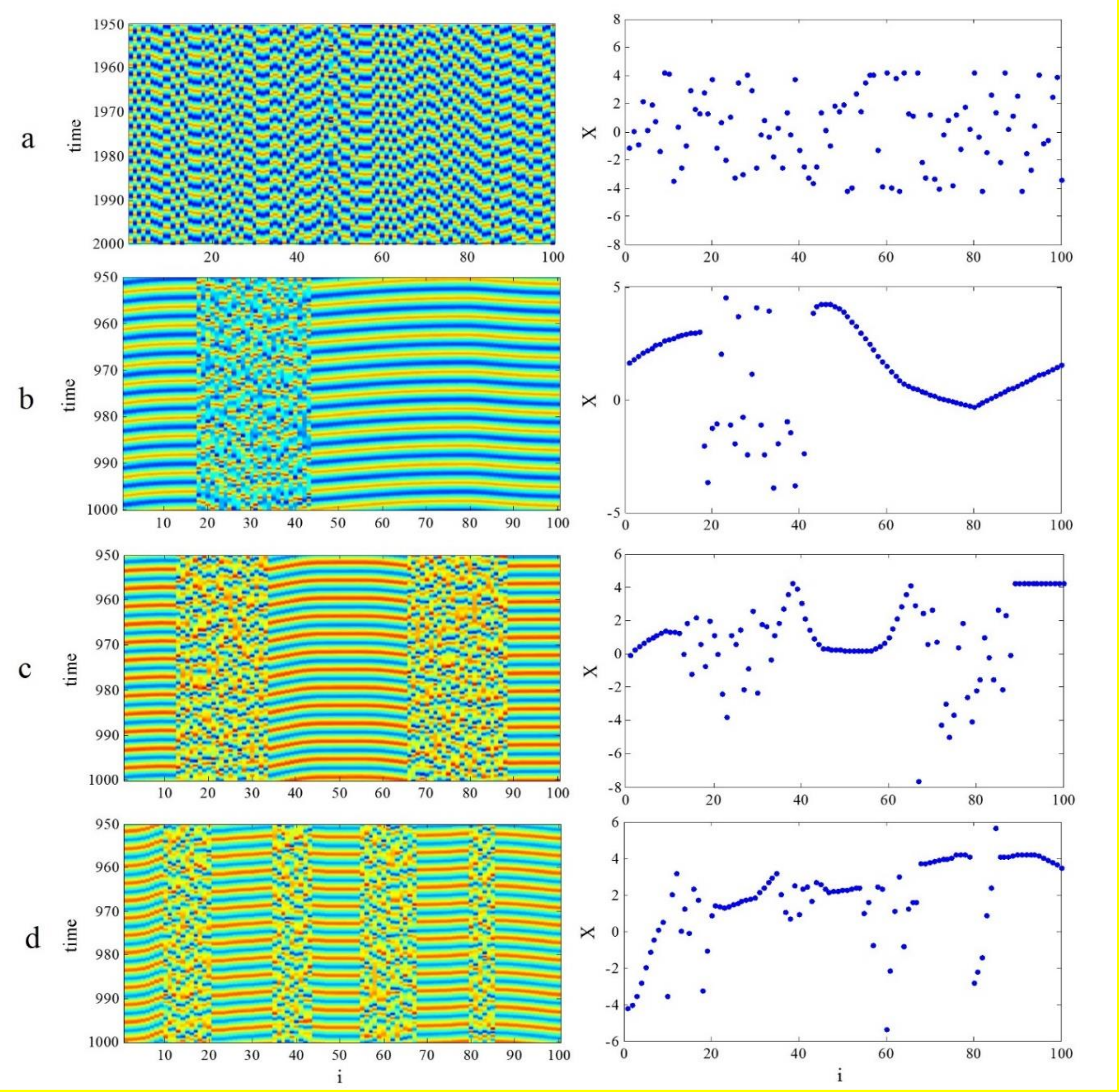
$$\dot{u}_i = -b y_i$$

where d is the coupling parameter, a = 7, b = 0.1.

3. Formation of perfect and imperfect chimeras

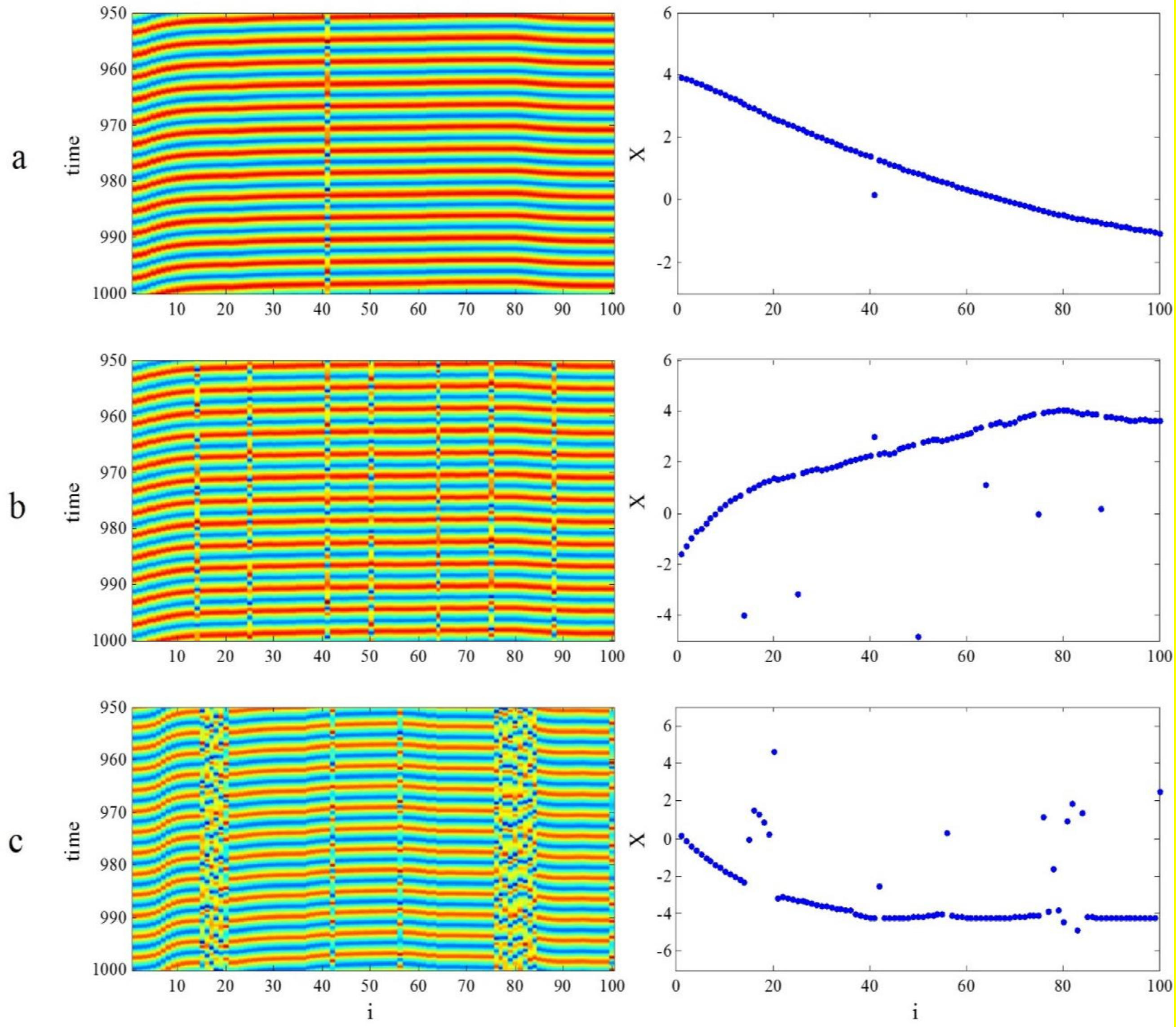
Let us see what happens as the coupling parameter d is increased:

Fig. 3
(a) Asynchronous state of limit cycles at d=0.002, (b) a perfect one -headed chimera state at d=0.005, (c) a perfect 2-headed chimera with d=0.007, and (d) a perfect 4 –headed chimera with d=0.009, all for random initial conditions, x in [−15,15], z in [−30,30] in the the x-z plane.



How did this happen?

Fig. 4. Select 1 or more "rebel" particles closer to the strange attractor:
(a) Imperfect synchronization with 1 "rebel",
(b) imperfect synchronization with 7 "rebels",
for d = 0.005.
(c) imperfect chimera for
d = 0.01.



4. An analytical calculation for the central limit cycle

Let us now attempt to obtain analytically an approximate form of our limit cycle by substituting in (1) the expressions:

$$\hat{x}(t) = A_1 \cos(\omega t) + A_2 \sin(\omega t), \qquad \hat{y}(t) = B_1 \cos(\omega t) + B_2 \sin(\omega t),$$
$$\hat{z}(t) = C_1 \cos(2\omega t) + C_2 \sin(2\omega t), \quad \hat{u}(t) = D_1 \cos(\omega t) + D_2 \sin(\omega t)$$

where we have omitted all higher terms of the form sin(kωt), cos(kωt), $k \geq 3$. We easily find the following results for the oscillation amplitudes:

$$D_1\omega = bB_2, \qquad D_2\omega = -bB_1, \qquad A_2\omega = B_1 - A_1 \quad , \; -A_1\omega = B_2 - A_2$$

$$-\omega B_1 = \frac{A_2C_1}{2} - \frac{A_1C_2}{2} + D_2, \qquad \omega B_2 = -\frac{A_1C_1}{2} - \frac{A_2C_2}{2} + D_1 \;, \qquad A_1 = \frac{B_1 - \omega B_2}{1+\omega^2}, \qquad A_2 = \frac{B_2 + \omega B_1}{1+\omega^2}$$

$$C_1 = \frac{2(b-\omega^2)(2B_1B_2 + \omega(B_1^2 - B_2^2))}{\omega(B_1^2 + B_2^2)}, \qquad C_2 = \frac{2(b-\omega^2)(B_2^2 - B_1^2 + 2\omega B_1 B_2)}{\omega(B_1^2 + B_2^2)}$$

$$B_1^2 + B_2^2 = 8(\omega^2 - b)(1+\omega^2), \qquad B_1^2 + B_2^2 = 2a(1+\omega^2)$$

From the last two equations we now derive the very important expression for the frequency of oscillations:

$$\omega^2 = b + \frac{a}{4}$$

$$\omega^2 = b + \frac{a}{4}$$

We have tested the accuracy of these formulas against our numerical results and have found the results:

1) For the single oscillator: The frequency of the limit cycle is 1.6, while our estimate gives ω = 1.36, and the amplitudes of x, y, z, u are 4.56, 9.01, 7.7 and 0.83, while the analytically computed values are 3.74, 6.32, 5.91 and 0.46 respectively.

These are good order of magnitude estimates, but more importantly:

2) Our analytical frequency estimates approximately as well the frequency of the synchronized state, at least for coupling constants *d* = 0.005 and *d* = 0.01. For example, when all particles have a frequency close to ω = 1.6 our analytical formula gives ω = 1.36 .

THE IMPORTANCE OF ASYMMETRY IN NONLINEAR OPTICS

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Outline

- Background & Motivation
- The Asymmetric Active Coupler (AAC) of 2 Waveguides
  Stable Nonlinear Supermodes (NS), Modulational Stability
  and Directed Transport in coupled waveguides (Couplers)
- Coupled, Asymmetric Semiconductor Lasers (ASL)
  Stable Asymmetric Phase Locked States, Hopf Bifurcations
  and Exceptional Points in Photonic Dimers (“meta-molecules”)
- Asymmetry in Gain and Loss provides new experimental
  opportunities in Non – Hermitian Photonics
- Conclusions and Future Outlook

Active Coupler (PT - symmetric)

- The presence of gain and loss makes the dynamics of the system non-Hermitian and in general non-integrable

- In the special case where the coupler is PT-symmetric, there exist two invariants of motion, rendering the system integrable, despite the fact that the total power S_0 is not conserved

- The dynamics of the system is reciprocal with respect to the direction of light

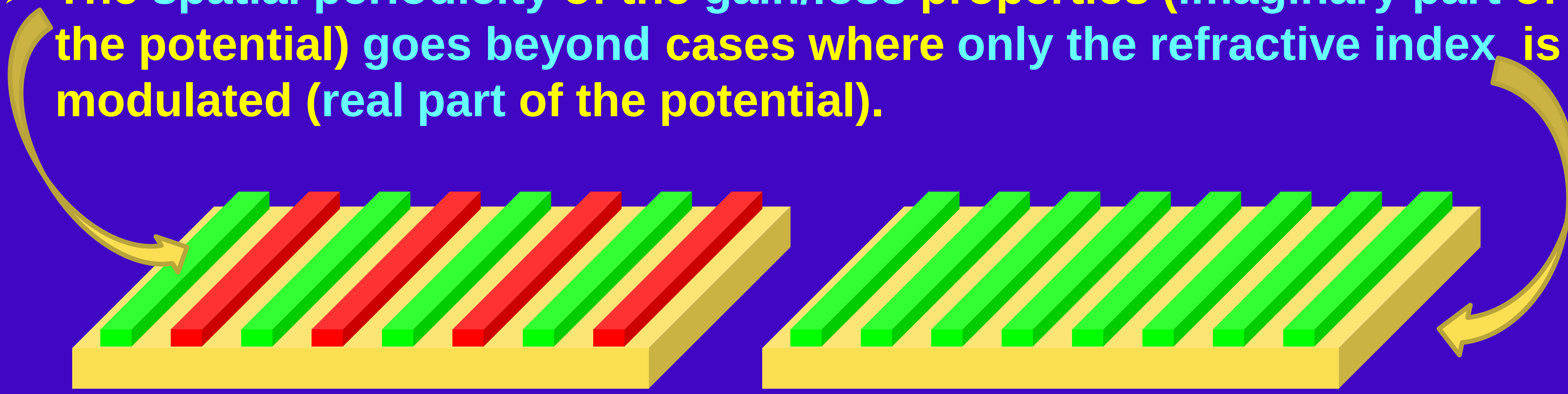
However:

- No finite-power fixed points (Nonlinear Supermodes) exist!

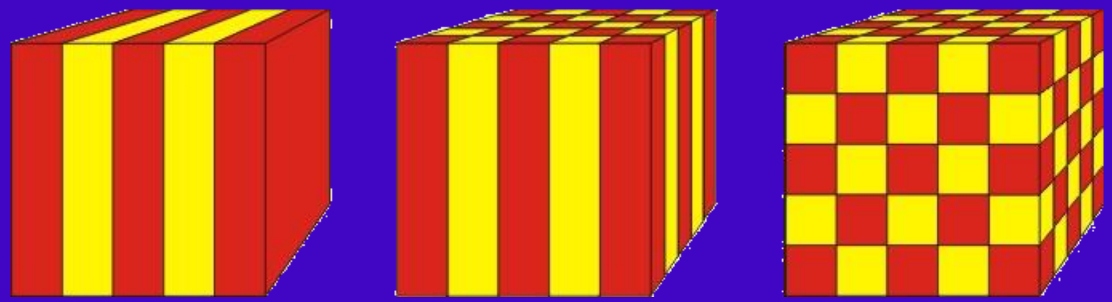
- Exceptional points arise only for $\Delta=0$ detuning in the linear spectra where complex eigenvalues and eigenvectors collapse to lower dimension and can lead to a 10^3 amplification of a preferred mode!

Background & Motivation

- The spatial periodicity of the gain/loss properties (imaginary part of the potential) goes beyond cases where only the refractive index is modulated (real part of the potential).

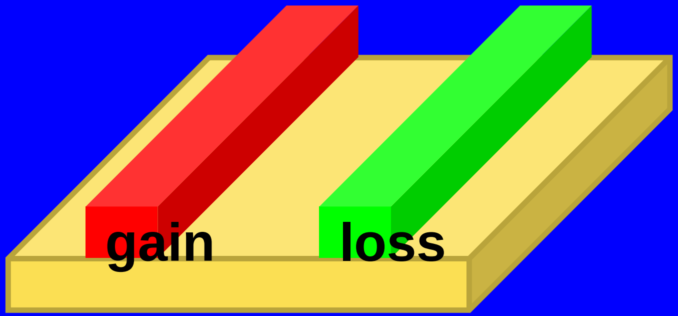


- Photonic structures are based on the engineering of the material properties in order to provide functionality required for photonic circuitry and integrated photonics.



The Asymmetric Active Coupler (AAC)

- The fundamental Active Coupler possesses non-reciprocal dynamics and exhibits directed power transfer in the case of PT-symmetric gain-loss.



- PT - symmetry:
  identical waveguides → refractive index is symmetric
  → gain in one = loss in the other

- However, the PT - symmetric active coupler does not have finite power Nonlinear Supermodes!

The Asymmetric Active Coupler (AAC)

The modal amplitudes of the two waveguides are governed by the Coupled Mode Equations:

$$-i\frac{dA_1}{dz} = (\beta_1 + i\alpha_1)A_1 + \gamma\left(|A_1|^2 + \sigma|A_2|^2\right)A_1 + \frac{\kappa}{2}A_2$$

$$-i\frac{dA_2}{dz} = (\beta_2 + i\alpha_2)A_2 + \gamma\left(\sigma|A_1|^2 + |A_2|^2\right)A_2 + \frac{\kappa}{2}A_1$$

$\beta_j + i\alpha_j$: complex propagation constant of the waveguide j

$\alpha_j > 0\ (< 0)$: loss (gain)

$\kappa > 0$: linear coupling coefficient

$\gamma, \sigma > 0$: nonlinear parameters

Y. Kominis, T. Bountis and S. Flach, Scientific Reports 6: 33699 (2016).

Stokes variables

It is convenient to introduce the Stokes Variables:

$$\boxed{\begin{aligned} S_0 &= |A_1|^2 + |A_2|^2 \\ S_1 &= |A_1|^2 - |A_2|^2 \\ S_2 &= A_1^* A_2 + A_1 A_2^* \\ S_3 &= i(A_1^* A_2 - A_1 A_2^*) \end{aligned}}$$

S_0 : total power of the coupler

S_1 : deviation from power balance

$S_0^2 = S_1^2 + S_2^2 + S_3^2 \quad \uparrow$ the dynamics of the system is 3-dimensional, described by the Stokes vector $\vec{S} \equiv (S_1, S_2, S_3)$

Coupled Mode Equations

Coupled Mode Equations:

$$\frac{dS_0}{dz} = -\delta S_0 - \alpha S_1$$

$$\frac{dS_1}{dz} = -\alpha S_0 - \delta S_1 + \kappa S_3$$

$$\frac{dS_2}{dz} = -\delta S_2 - (\beta + \chi S_1) S_3$$

$$\frac{dS_3}{dz} = -\delta S_3 + (\beta + \chi S_1) S_2 - \kappa S_1$$

Parameters:

$$\alpha = \alpha_1 - \alpha_2$$

$$\delta = \alpha_1 + \alpha_2$$

$$\beta = \beta_1 - \beta_2$$

$$\chi = \gamma(1 - \sigma)$$

- **We consider cases where $\alpha_1\alpha_2<0$ (gain and loss) so that the sign of the parameter $\alpha=\alpha_1-\alpha_2$ determines whether the first waveguide has loss and the second has gain ($\alpha>0$) or vice versa ($\alpha<0$) .**
- **The crucial parameters $\delta = \alpha_1+\alpha_2$ and $\beta = \beta_1-\beta_2$ determine the excess gain/loss and the asymmetry that quantifies the deviation from the PT symmetry point at which $\delta = \beta = 0$.**

Existence of Nonlinear Supermodes

- We first note that there exists always a trivial zero fixed point *O* for which $S_1=S_2=S_3=0$.

- The introduction of asymmetry allows for the existence of fixed points of the system which correspond to finite-power, constant-intensity Nonlinear Supermodes (NS) of the AAC.

- These supermodes represent optical fields that propagate unchanged along the coupler despite the presence of gain, loss, asymmetry and nonlinear effects.

- The utilization of the normalized Stokes vector $\vec{F} = \vec{S} / S_0$ allows for visualization of the dynamics on a Bloch Sphere of unit radius.

Location of Nonlinear Supermodes

$$F_1^{(0)} = -\Delta$$

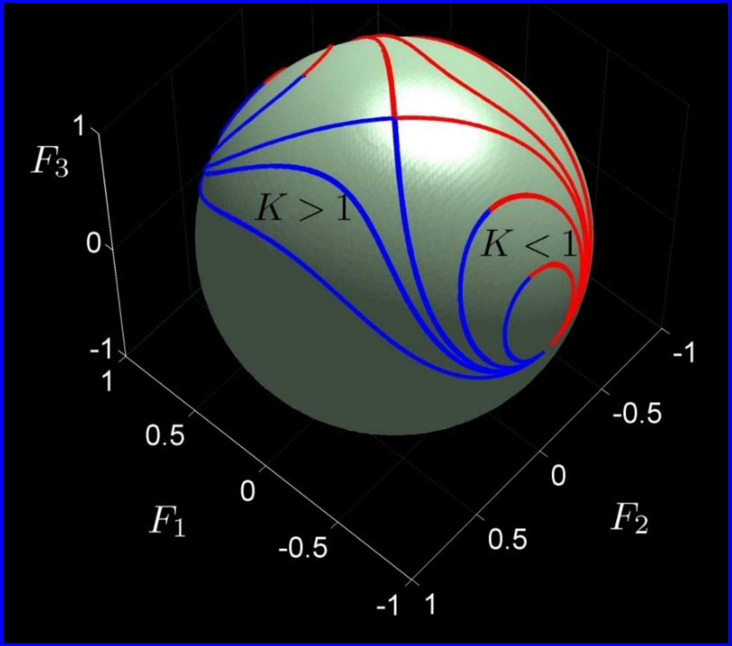
$$F_2^{(0)} = m\frac{K}{2}\sin\varphi$$

$$F_3^{(0)} = \frac{K}{2}\left(1-\cos\varphi\right)$$

$\Delta \equiv \delta / \alpha, \quad K \equiv \kappa / \alpha, \quad X \equiv \chi / \alpha$

$$\tan\left(\varphi / 2\right) \equiv |\Delta / \Lambda_{\pm}| = \sqrt{(1-\Delta^2) / (K^2 + \Delta^2 - 1)}$$

$$\Lambda_{\pm} = \pm\Delta\sqrt{(K^2 + \Delta^2 - 1) / (1-\Delta^2)}$$



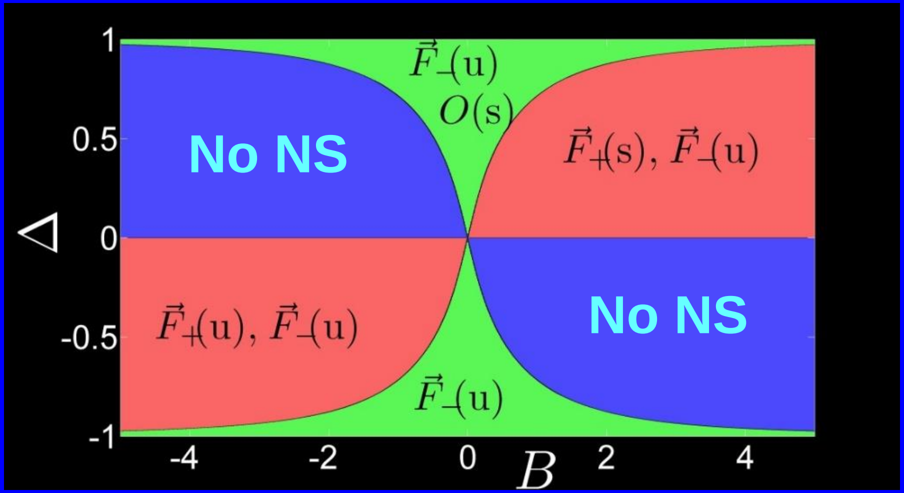
The location of the non-trivial NS on the Bloch sphere depends :

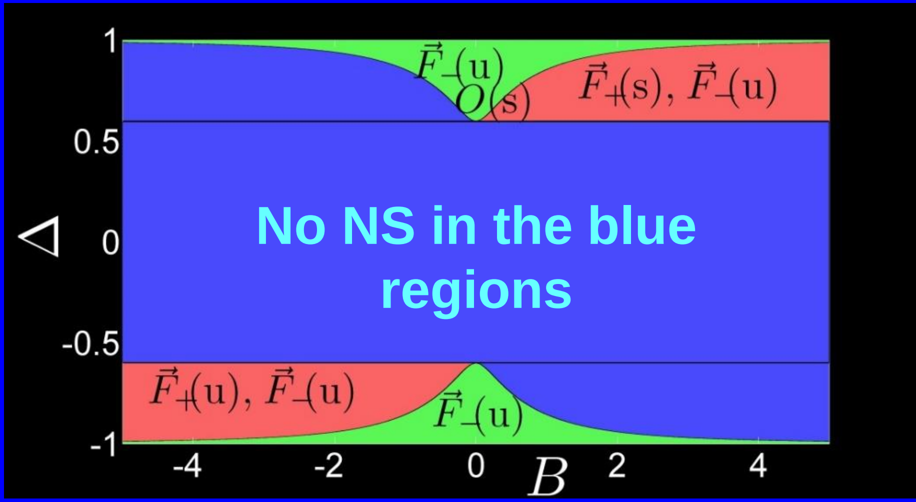
- on Δ and K
- on whether $K > 1$ or $K < 1$

➢ All curves touch at their common points: $\vec{F} = (\pm 1, 0, 0)$

➢ Two of the NS are symmetric with respect to the plane F_2=0

Existence and Stability of NS





The domains of existence of stable (s) and unstable (u) NS of the AAC in the (Δ, B) parameter space for α =1 and X=1. (left) K=1.2, (right) K=0.8.

The zero fixed point O exists for all parameter values but it is stable only in the regions marked with O(s).

$\Delta = (\alpha_1 + \alpha_2) / (\alpha_1 - \alpha_2)$

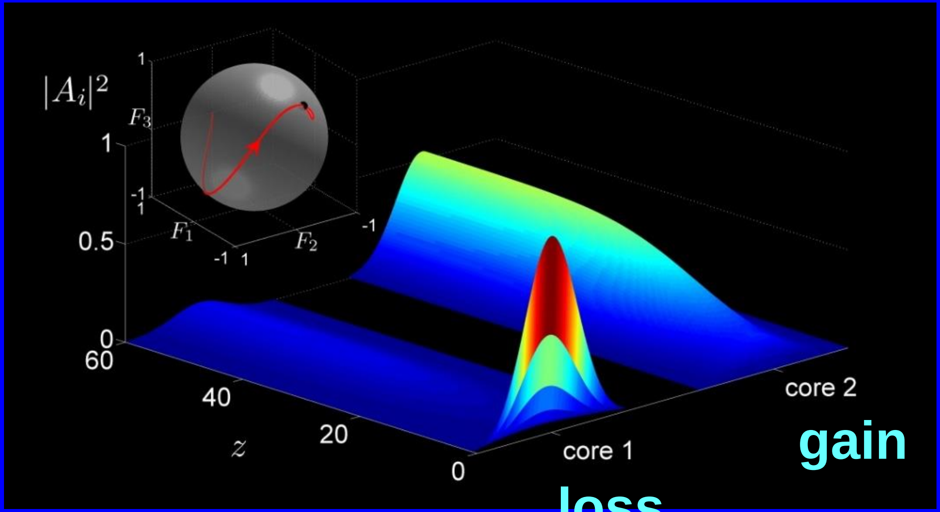
degree of gain / loss asymmetry

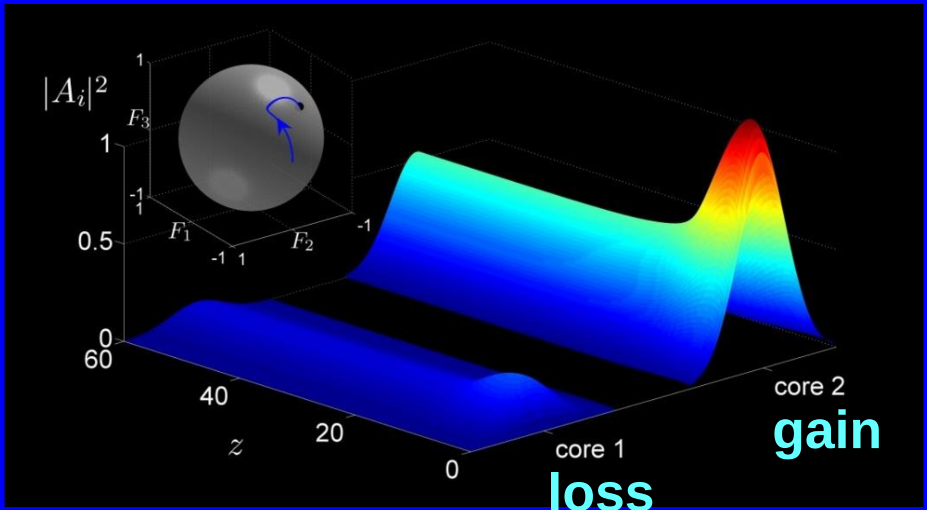
$B = (\beta_1 - \beta_2) / (\alpha_1 - \alpha_2)$

degree of geometric asymmetry

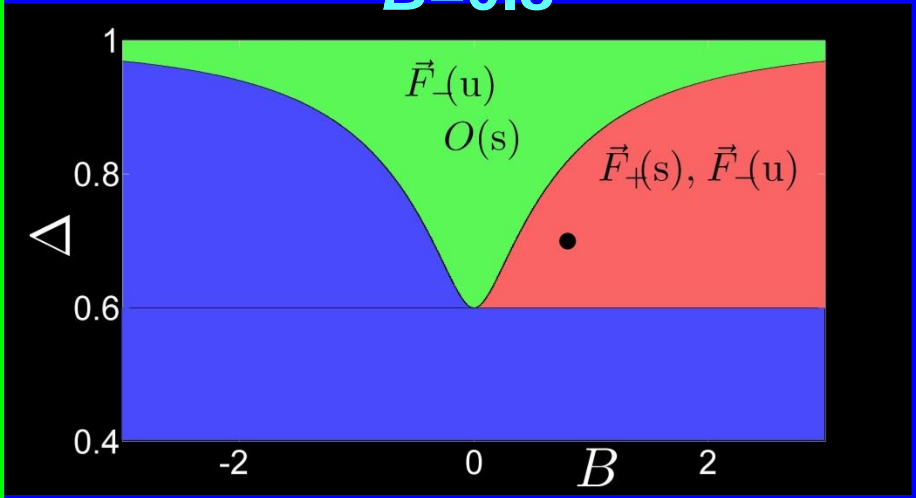
PT - symmetry implies $\Delta = B = 0$

Stable AAC Dynamics (α=1, X=1, K=0.8, Δ=0.7)





B=0.8



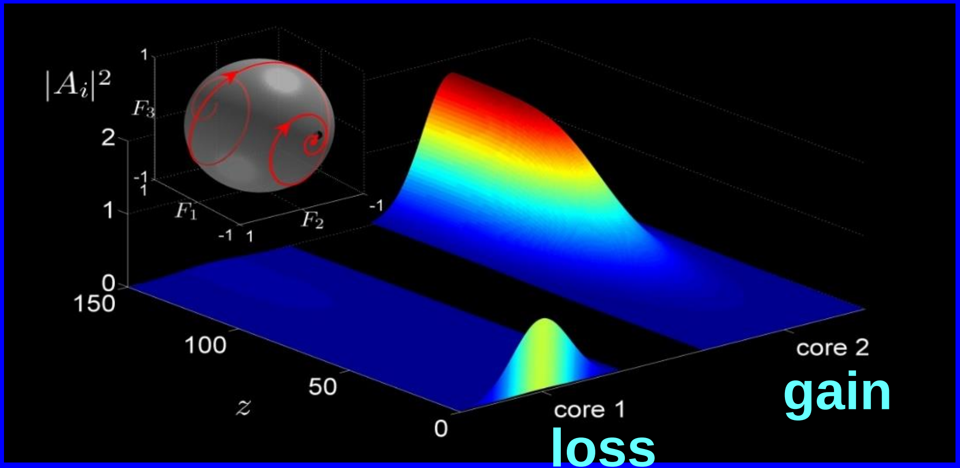
- **Existence of a Stable Nonlinear Supermode implies:**
- **Directed Power Transport to a finite-power final state with desired total power and amplitude ratio :**

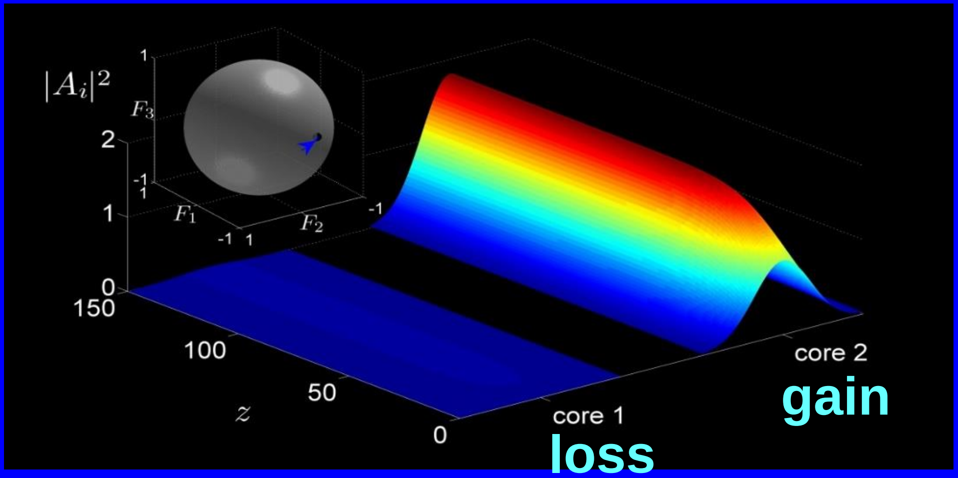
$|A_1|^2 / |A_2|^2 = (1-\Delta) / (1+\Delta) = 0.18$

Total Power : $S_{0,+} = 0.64$

Far from P-T Symmetry ($\Delta \simeq 1$)

$\alpha=2$, $X=1$, $K=0.8$, $\Delta=0.95$, $B=4$





- Existence of a Stable Nonlinear Supermode

Amplitude Ratio :

$$|A_1|^2 / |A_2|^2 = (1-\Delta)/(1+\Delta) = 0.026$$

Initial conditions evolve to a final state where the total power is finite and located almost exclusively in the second waveguide.

- Even small excitation in the first waveguide directs all power to the stable NS with all power on the second waveguide.

Transversal Modulational Instability

The electric field envelopes of the two waveguides are governed by Coupled Mode Equations of the Nonlinear Schrodinger type:

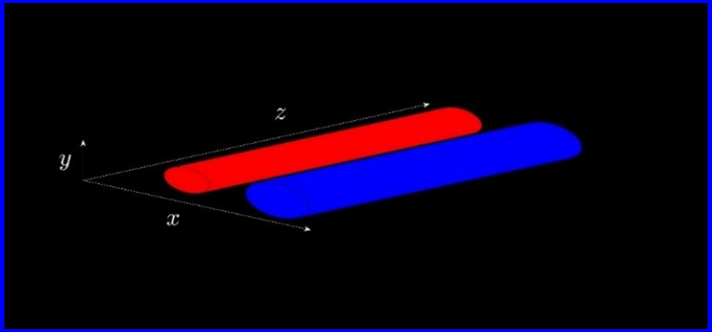
$$i\frac{\partial u_{1,2}}{\partial z}+\frac{\partial^2 u_{1,2}}{\partial x^2}+(\beta_{1,2}+i\alpha_{1,2})u_{1,2}+$$
$$+\gamma\left(|u_{1,2}|^2+\sigma|u_{2,1}|^2\right)u_{1,2}+\frac{\kappa}{2}u_{2,1}=0$$

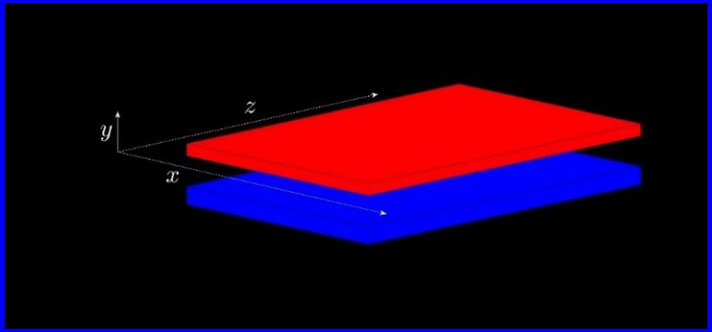
where

$$u_i = A_i \exp(ibz),\ i=1,2,$$

and x represents the transversal direction

(see below)





Y. Kominis, T. Bountis and S. Flach, *Phys. Rev. A* 95, 063832 (2017).

AAC subject to Modulational Instability

The constant amplitudes $\boxed{A_i = |A_i| \exp(i\varphi_i), \ i = 1,2,}$ are those of the electric field envelopes of the NS discussed earlier:

$$\boxed{|A_1|^2 = \frac{\beta_1\alpha(\beta - 1) + \kappa / 2(\sqrt{\alpha}(\alpha - 1)\cos\varphi}{\gamma(1-\sigma)(\alpha - 1)}, \quad |A_2|^2 = \frac{1}{\alpha}|A_1|^2}$$

where $\alpha = -\alpha_2/\alpha_1 > 0$, $\beta = \beta_2/\beta_1$, b and the $\varphi = \varphi_1 - \varphi_2$ are defined through equations determined by the existence of stable NS. To study MI we now consider small perturbations

$$\boxed{u_{1,2} = (A_{1,2} + \varepsilon_{1,2})\exp(ibz), \qquad i = 1,2,}$$

and keeping only linear terms in the $\varepsilon_{1,2}$ we obtain the system:

$$i\frac{\partial \varepsilon_{1,2}}{\partial z} + \frac{\partial^2 \varepsilon_{1,2}}{\partial x^2} + F_{1,2}\varepsilon_{1,2} + H_{1,2}\varepsilon^*_{1,2} + M_{1,2}\varepsilon_{2,1} + G\varepsilon^*_{2,1} = 0$$

AAC subject to Modulational Instability

Consider now the behavior of Fourier modes of the form:

$$\varepsilon_{1,2} = c_{1,2}\exp i(\omega x - \lambda z) + c^*_{1,2}\exp(-i(\omega x - \lambda^* z))$$

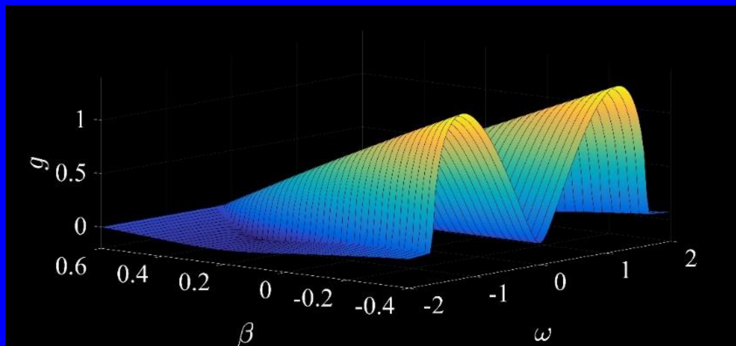
and solve the eigenvalue problem : $L\psi = \lambda\psi$, where:

$$L = \begin{pmatrix} \omega^2 - F_1 & -H_1 & -M & -G \\ H_1^* & \omega^2 + F_1^* & G^* & M^* \\ -M^* & -G & \omega^2 - F_2 & -H_2 \\ G^* & M & H_2^* & \omega^2 + F_2^* \end{pmatrix}$$

Clearly, instability occurs when there is an eigenvalue λ with a positive imaginary part. Thus, we search for stable cases, where λ has only negative imaginary part, $\mathrm{Im}(\lambda)=g < 0$.

Modulational Instability

(a)



(b)

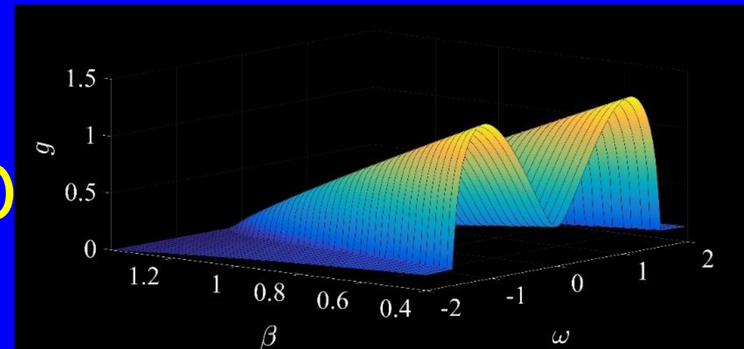
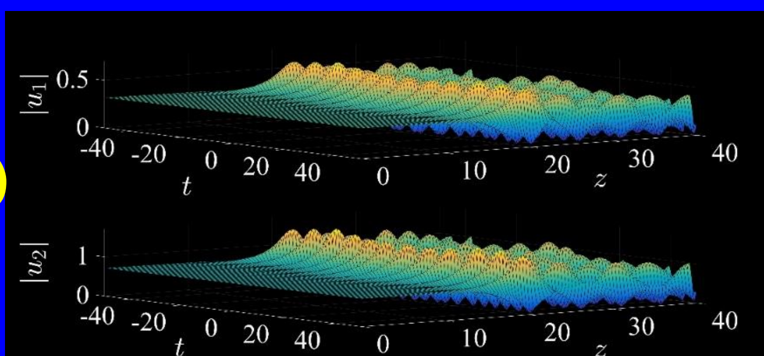


Fig. 1: Growth rate g =max(Imλ_i) i=1,2,3,4 (g>0 for instability) for the case of self-focusing nonlinearity (γ = 1 and σ = 0) for a gain-loss imbalance corresponding to α = 0.2 for the two Nonlinear Supermodes (a) and (b). Both NS are modulationally unstable.

(a)



(b)

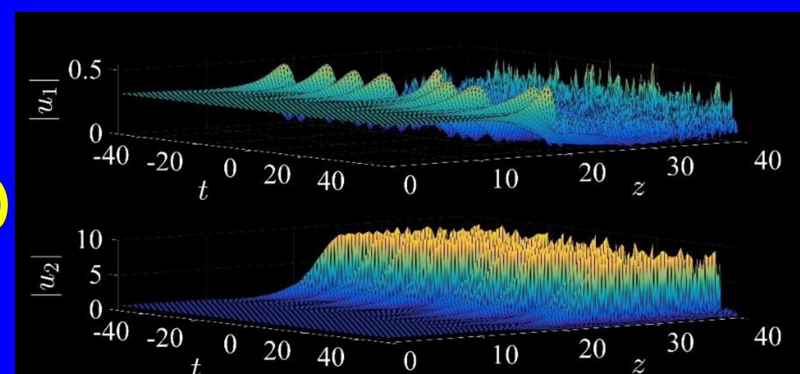


Fig. 2: Modulation Instability of the NS under a random noise perturbation of order 10^{-2} superimposed at z = 0, for the cases of Fig. 1(a) with β = 0.2 and Fig. 1(b) with β = 1

Modulational Stability

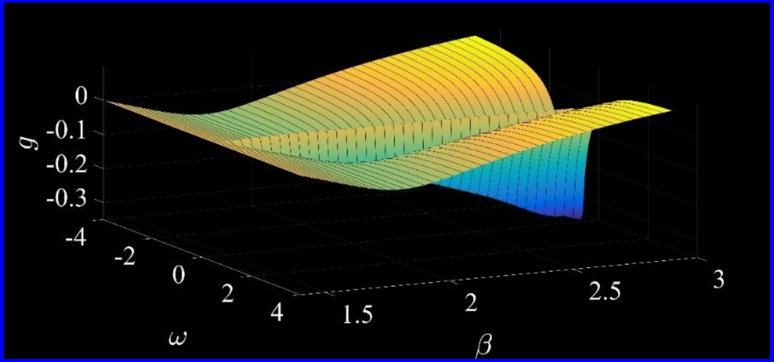
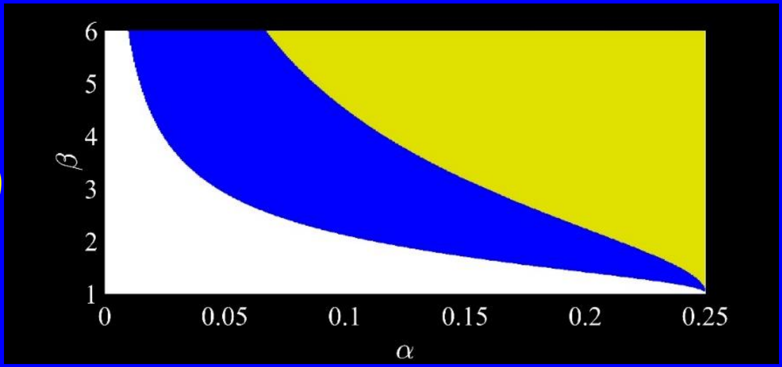


Fig. 3: Growth rate g for a self-defocusing nonlinearity ($\gamma = -1$) and $\sigma = 0$ for a gain-loss imbalance corresponding to $\alpha = 0.2$ and an NS that is stable ($g < 0$ for all ω) for a range of parameter values.

(a)



(b)

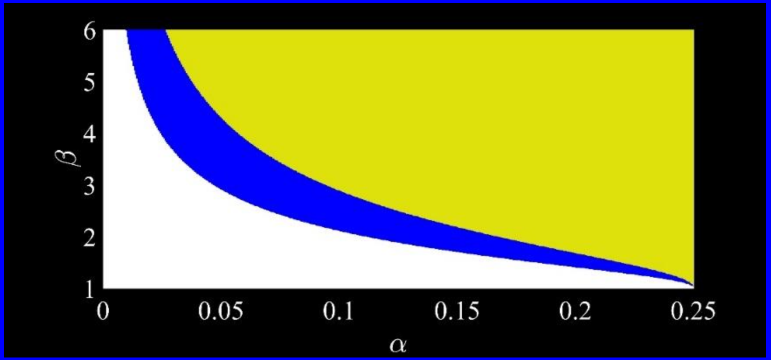
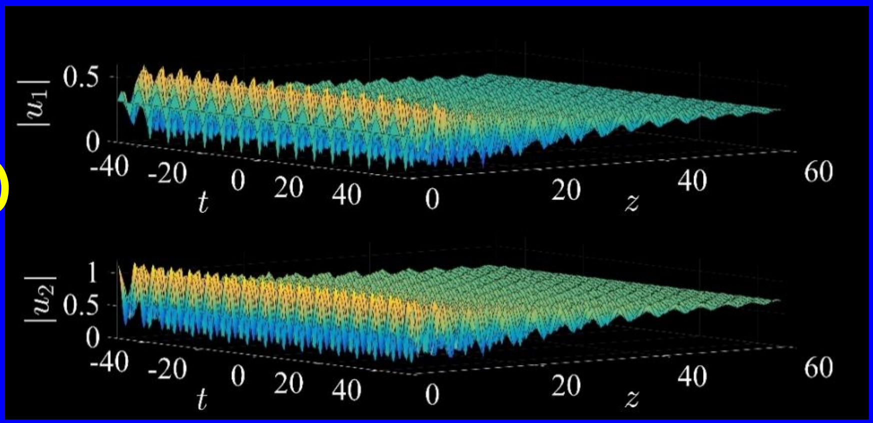


Fig. 4: The NS are modulationally stable against perturbations (with $\omega = 0$) in the yellow area and modulationally stable with arbitrary ω in the blue area. (a) corresponds to $\sigma = 0$ and (b) to $\sigma = 0.5$.

Modulational Stability

(a)



(b)

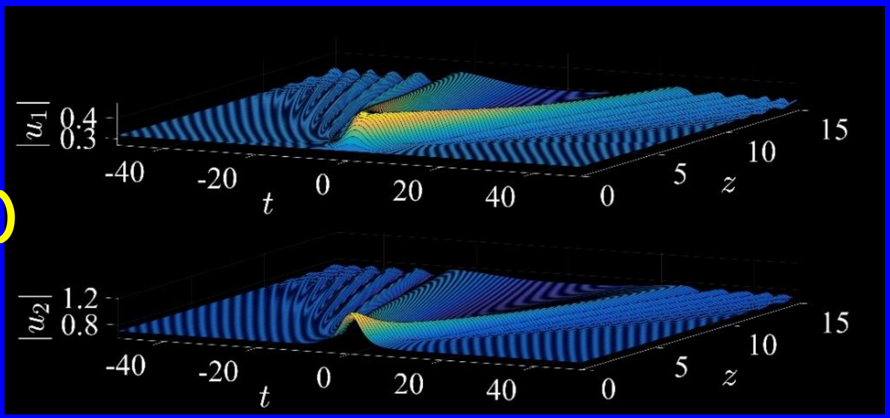


Fig. 5: Wave evolution of periodic and localized perturbations superimposed on a modulationally stable Nonlinear Supermode. Parameter values as in Fig. 3(a), and β = 1.8. In the second waveguide, a spatially periodic wave u = 0.5 cos(t) in panel (a) and a spatially localized wave u = 0.5sech(0.5t) in panel (b) have been superimposed on the stable NS at z = 0.

Coupled Semiconductor lasers

- Coupled semiconductor lasers are photonic structures with great potential for many applications in optical communications. A pair of coupled lasers is said to represent a “photonic molecule".

- We investigate nonlinear, asymmetric coupled lasers with carrier density dynamics and study the existence and stability of asymmetric phase-locked modes .

- We discover stable phase-locked modes with arbitrary power, amplitude ratio and phase difference, which for appropriate pumping and detuning bifurcate to stable limit cycles (Hopf bifurcations).

- We study the zero mode under asymmetry and discover lines in parameter space with exceptional points and other types of bifurcations, including laser termination at a stable zero state.

Coupled Asymmetric Semiconductor lasers: Phase locked states and Hopf Bifurcations

Y. Kominis, A. Bountis and V. Kovanis, Phys. Rev. A, 96, 043836 (2017)

The modal amplitudes of the two lasers and the carrier densities are governed by the Coupled Ordinary Differential Equations (*):

$$\frac{dE_i}{dt} = (1 - i\alpha)E_i Z_i + i\eta\left(E_{i+1} + E_{i-1}\right) + i\omega_i E_i$$

$$T\frac{dZ_i}{dt} = P_i - Z_i - \left(1 + 2Z_i\right)|E_i|^2 \quad , \quad i = 1, 2, \ldots, M$$

where:

E_i : The electric fields , Z_i = The normalized excess carrier densities

α : Linewidth enhancement factor η : normalized coupling constant

P_i :Normalized excess pumping rates ω_i : normalized frequency detuning

(*) Winful, H. and Wang, S., Appl. Phys. Lett. 53, 1894 (1988).

Coupled Semiconductor lasers: Phase Locked States

Scaling our variables and writing $E_i = X_i \exp(\theta_i)$ we solve analytically our equations and obtain exact Phase Locked States:

$$X_0^2 = \frac{\Omega\Lambda\sin\theta(\rho^2-1)}{\rho[(\rho^2-1)-4\Omega\Lambda\rho\sin\theta]}, \quad \theta = s\pi + \tan^{-1}\left[\frac{\rho^2-1}{\alpha(\rho^2+1)}\right], \quad s = 0,1$$

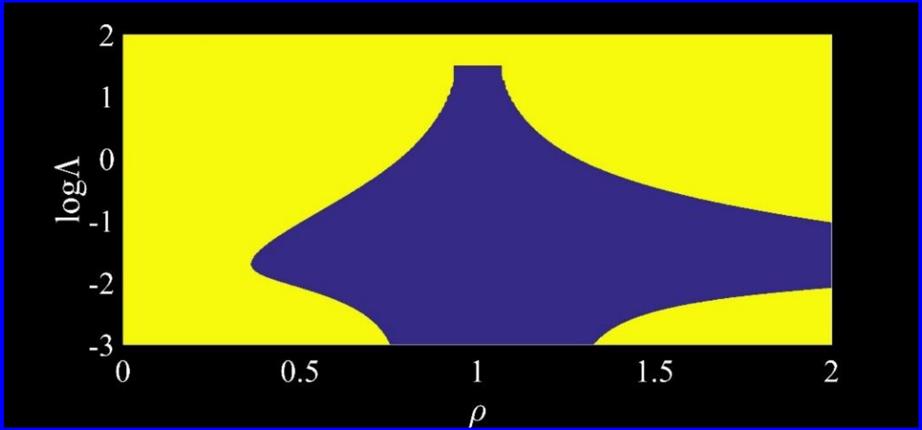
$$Z_1 = \Lambda\rho\sin\theta, \quad Z_2 = -\Lambda\cos\theta/\rho, \quad P_1 = X_0^2 + (1+2X_0^2)\Omega\Lambda\rho\sin\theta$$

$$P_2 = \rho^2 X_0^2 - (1+2\rho^2 X_0^2)\Omega\Lambda\sin\theta/\rho$$

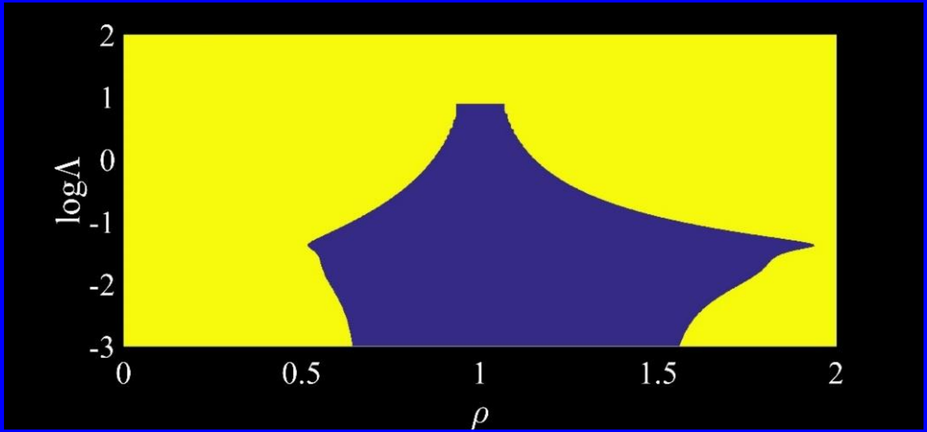
with $\theta = \theta_2 - \theta_1$, $X_0 = X_1$ and $\rho = X_2 / X_1$ (here $\Delta = 0$).

First important result: *Even in the case of symmetric lasers: $P_0 = P_1 = P_2$ and zero detuning $\Delta = \Omega_2 - \Omega_1 = 0$,* **we find asymmetric stable solutions!**

Asymmetric states for Identical Lasers Δ=0, P_1=P_2



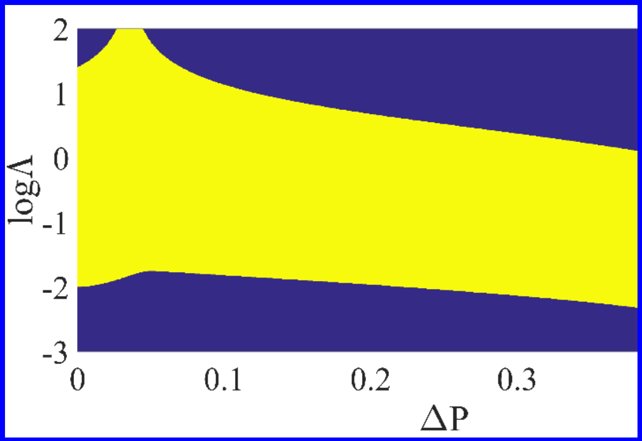
(a)



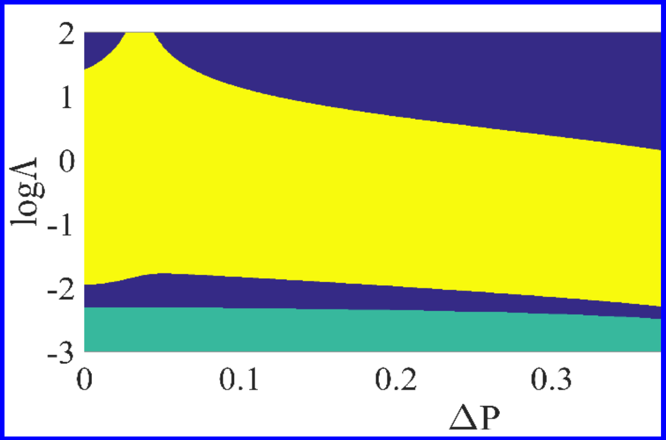
(b)

Fig. 6. Stable asymmetric phase-locked states in the (Λ,ρ) parameter space of identical lasers ($\Delta P = P_1 - P_2 = 0$). Blue and yellow areas correspond to stability and instability, respectively. (a) α = 5 and T = 400, (b) α = 1.5 and T = 400.

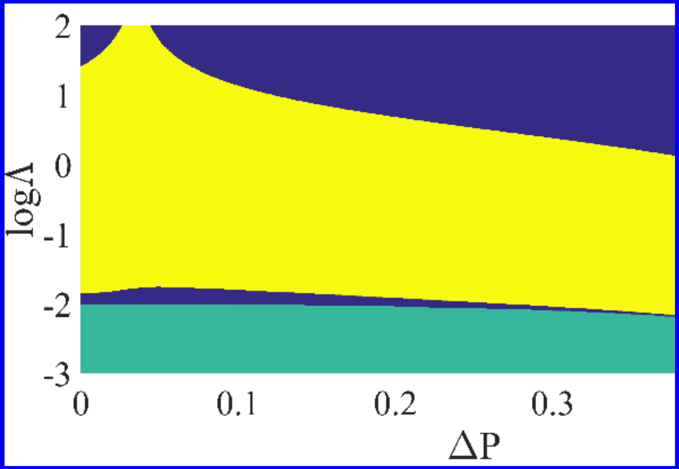
Asymmetric Lasers yield Hopf bifurcations



(a)



(b)



(c)

Fig. 7. Existence and stability regions of phase-locked states of asymmetric lasers in (Λ,ΔP) space. Yellow areas correspond to instability. At the boundaries of the dark blue stable regions Hopf bifurcations to stable limit cycles occur. The green area corresponds to nonexistence of a phase-locked state due to non-zero detuning. (a) $\Delta = 0$, (b) $\Delta = 0.05$, (c) $\Delta = 0.1$.

Stable and Unstable Phase Locked States

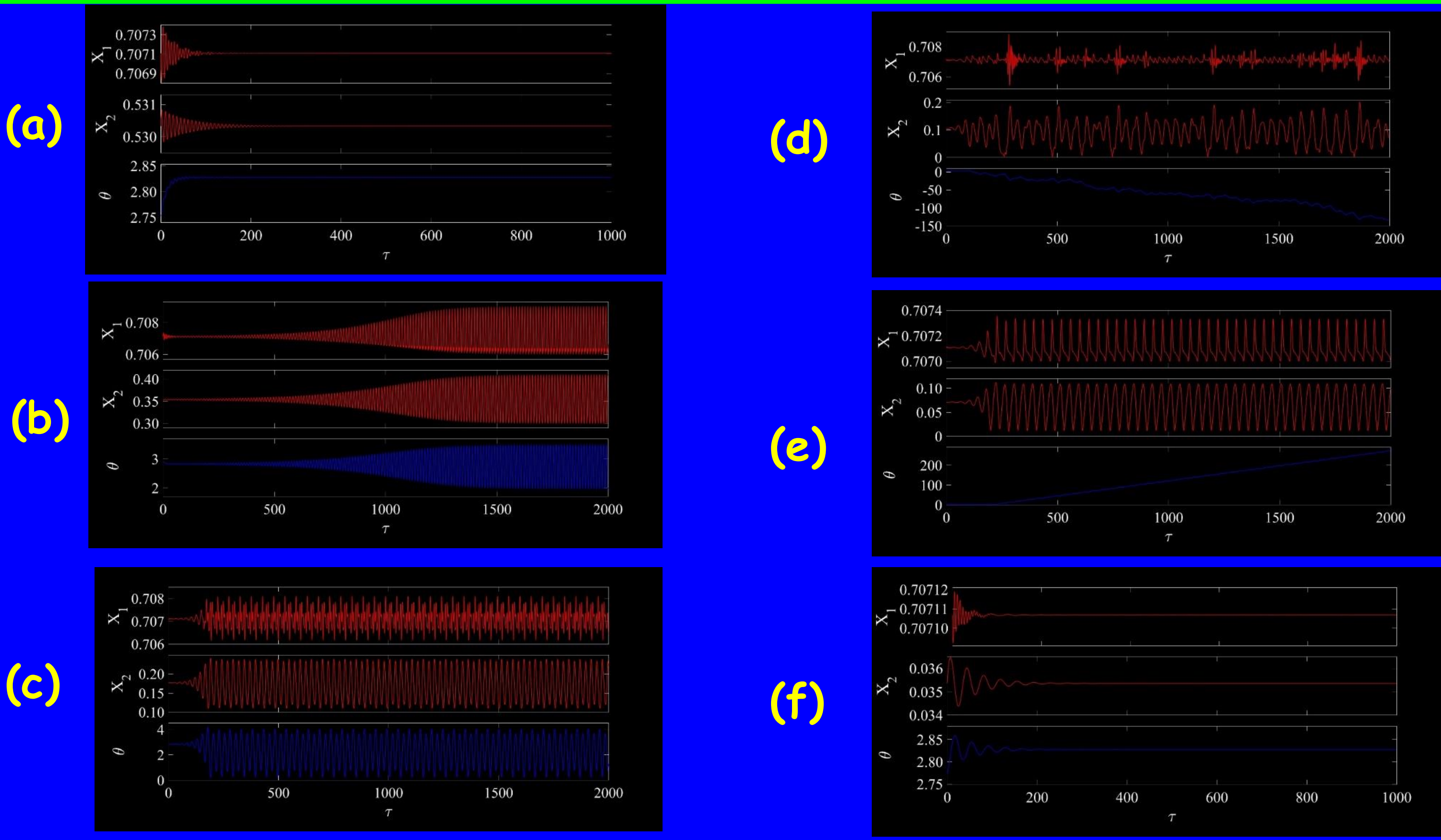


Fig. 8. Time evolution of the electric field amplitudes and phase differences. We start with asymmetric phase-locked states with $\theta = 0.9\pi$ and $\rho = 0.75$ (a) to $\rho = 0.05$ (f), perturbed by random noise. Stable phase-locked states are in (a), (f). Unstable phase-locked states evolve either to stable limit cycles [(b), (c), (e)] or to chaotic states (d).

The Zero State: A line of Exceptional Points

We now examine the same system for $\alpha = 5$, $T = 400$, $P = 0.5$, and consider the zero state solution:

$$E_i = 0 \text{ and } Z_i = P_i / \Omega$$

Linearizing about this state, we obtain the eigenvalues:

$$\lambda_{1,2} = \Lambda\left\{\bar{P} + i(\bar{\Omega} - \alpha\bar{P}) \pm \sqrt{[\Delta P + i(\Delta - \alpha\Delta P)]^2 - 1}\right\},$$

$$\lambda_{3,4} = -1/2P$$

Exceptional Points at $\Delta - \alpha\,\Delta P = 0$!!

where we define:

$$\bar{P}_i = (P_1 + P_2)/2\Lambda, \quad \bar{\Omega} = (\Omega_1 + \Omega_2)/2\Lambda,$$

$$\Delta P = (P_1 - P_2)/2\Lambda, \quad \Delta = (\Omega_1 + \Omega_2)/2\Lambda$$

The line $\Delta - \alpha\,\Delta P = 0$ in the parameter space generates important so – called Exceptional Points and generalizes PT-symmetric dimers, where spectral transitions occur only for zero detuning $\Delta = 0$!

Y. Kominis, A. Bountis and V. Kovanis, Phys. Rev. A 96, 053837 (2017)

Coupled Semiconductor lasers: Exceptional Points

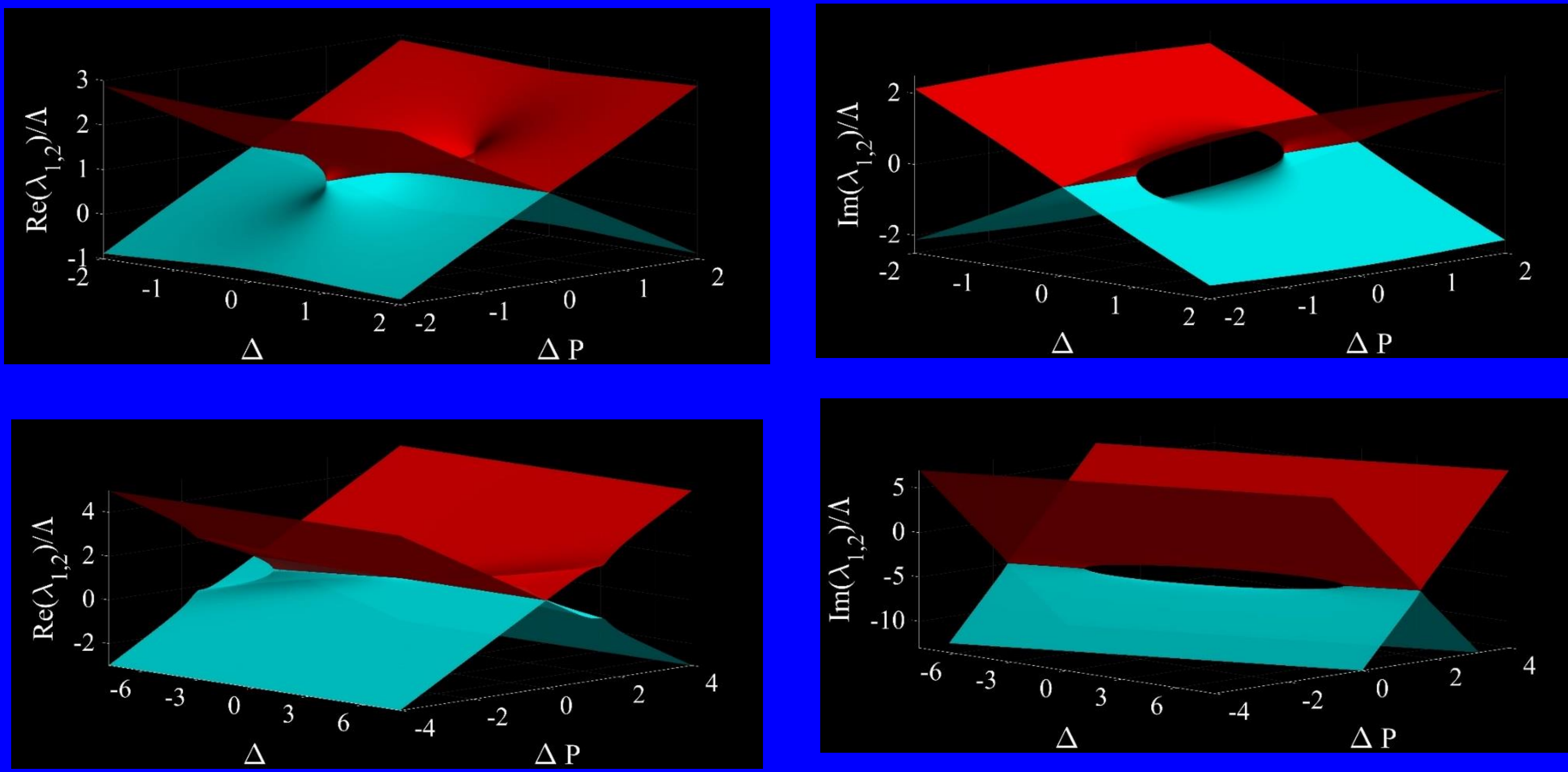


Fig. 9. Real (left) and imaginary (right) part of the normalized eigenvalues of the zero state for $\alpha = 0$, top panels, and $\alpha = 5$, bottom panels. Spectral transitions and exceptional points existence occur along the line $\Delta - \alpha\,\Delta P = 0$. Thus, a non-zero α raises the restriction of zero detuning ($\Delta = 0$), for the existence of exceptional points.

Coupled Semiconductor Lasers: Laser Termination

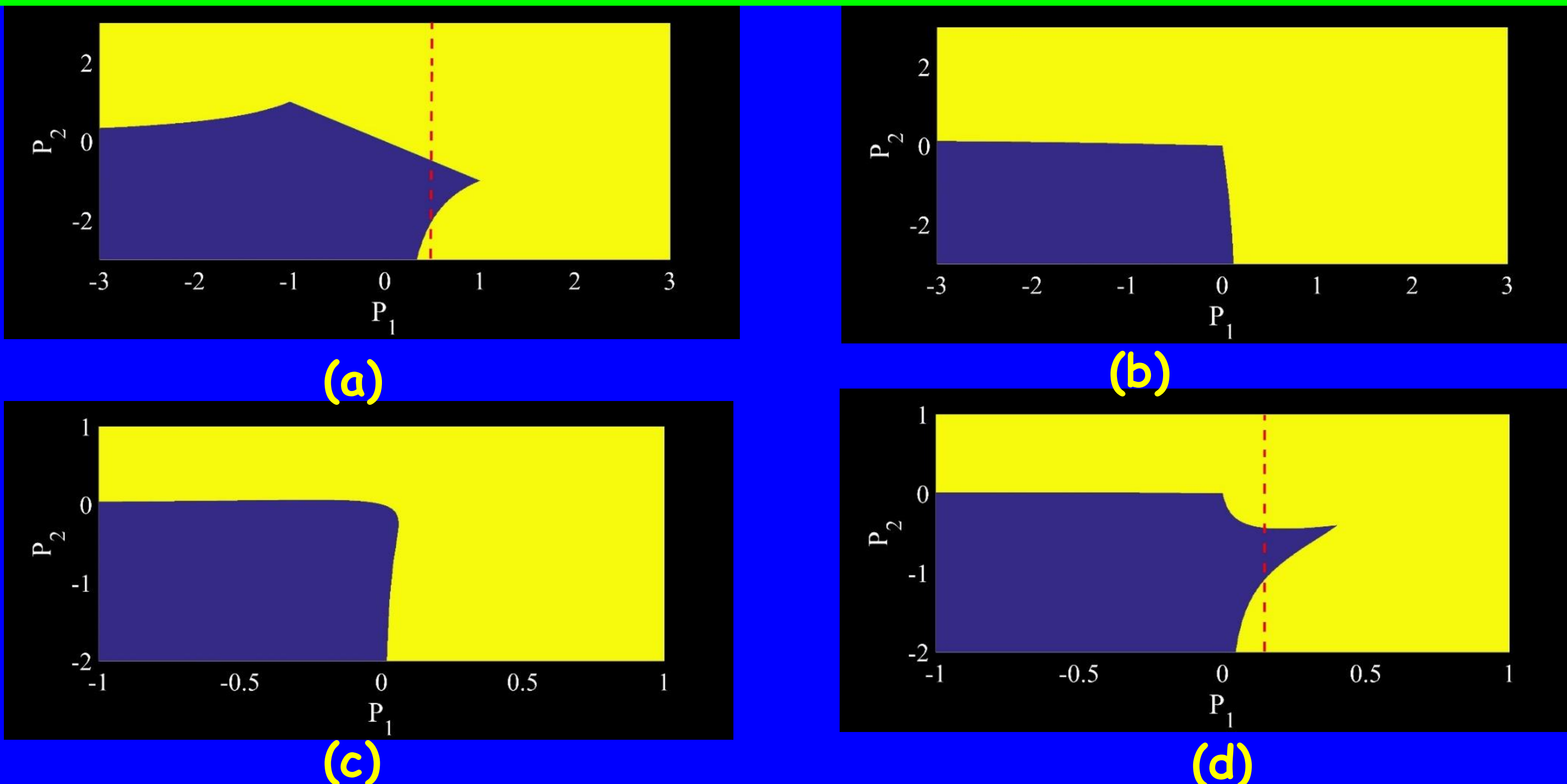


Fig. 10. Blue stability regions of the zero state in the (P_1, P_2) parameter space for $\alpha = 0$ (top panels), $\alpha = 5$ (bottom panels), $\Delta = 0$ (left) and $\Delta = 2$ (right) detuning. The red dashed lines correspond to $P_1 = 0.5$ (a) and $P_1 = 0.15$ (d). Increasing P_2 along the constant P_1 lines leads to laser death at a stable zero state!

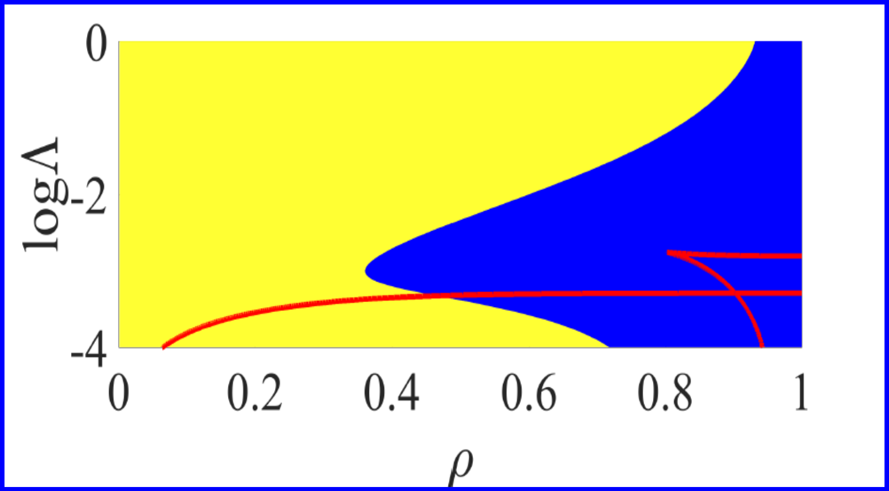
It is worth comparing these results with those obtained recently by other researchers [Z. Gao, M. T. Johnson and K. D. Choquette, arXiv:1801.0354, 2018] on P-T symmetric states.

We thus define a frequency detuning $\Delta\omega$, the gain contrast $\Delta\gamma$, and the net gain δ, as follows:

$$\boxed{\begin{aligned} \Delta\omega &\equiv \alpha(Z_2 - Z_1) - \Delta = \Lambda\cos\theta(\rho - 1/\rho) \\ \Delta\gamma &\equiv Z_2 - Z_1 = -\Lambda\sin\theta(\rho + 1/\rho) \\ \delta &\equiv Z_1 + Z_2 = \Lambda\sin\theta(\rho - 1/\rho) \end{aligned}}$$

The PT-symmetric case corresponds to $\Delta\omega = 0$, $\rho = 1$ and $\delta = 0$, i.e. zero net gain.

So let us see what new fruits we shall reap for our phase locked states in the case of asymmetry!



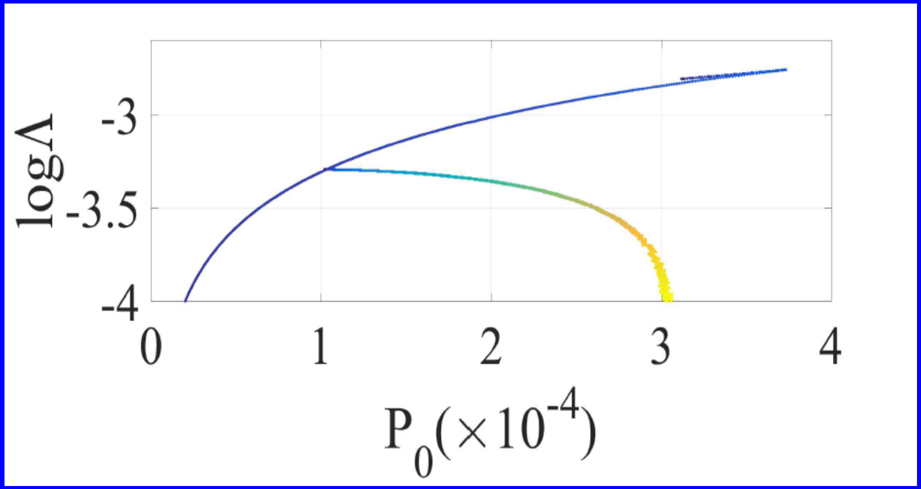
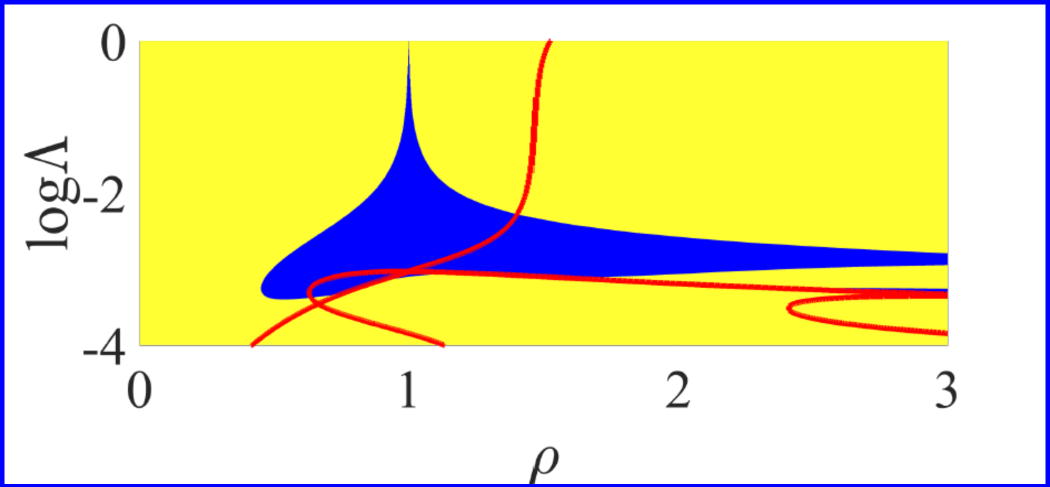


Fig. 12: Stability and location of Hopf Bifurcation and Exceptional Points in the Λ, ρ (above) and Λ, P_0 (below) for the case of zero frequency detuning (Δ = 0) and equal pumping ($P_1 = P_2 = P_0$).

Above: The Hopf Bifurcation Points are located at the boundary between stable (blue) and unstable (yellow) regions. Red lines depict the location of the Exceptional Points.

Below: The line colormap depicts the asymmetry of the respective phase-locked states (blue and yellow color correspond to ρ=1 and ρ < 1.



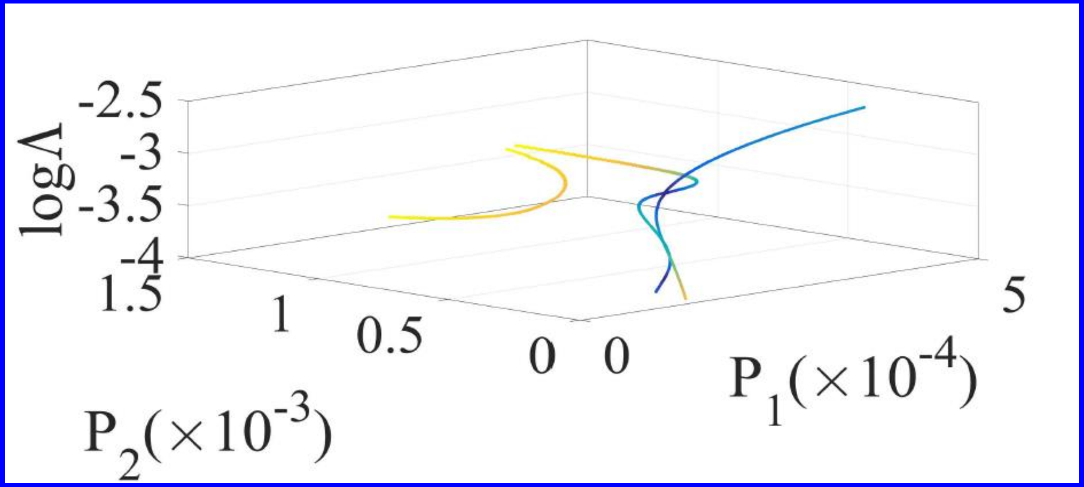


Fig. 13: Stability and location of Hopf Bifurcation and Exceptional Points in the Λ, ρ (above) and (Λ, P_1, P_2) (below) for the case of zero frequency detuning (Δ =0) and unequal equal pumping rates ($P_1 \neq P_2$).

Above: The Hopf Bifurcation Points are located at the boundary between stable (blue) and unstable (yellow) regions. Red lines depict the location of the Exceptional Points.

Below: The these are the s = 0 phase-locked states (blue and yellow color correspond to ρ = 1 and ρ < 1, ρ >1. Results for the value of X_0=0. 0.01259.

Conclusions

- For the Asymmetric Active Coupler (AAC) we discovered finite power Stable Nonlinear Supermodes, and new conditions for Modulational Stability and Directed Transport

- For Asymmetric Semiconductor Lasers (ASL) we obtained Stable Asymmetric Phase Locked States, Hopf Bifurcations and Exceptional Points yielding new types of bifurcations

- Thus, asymmetric photonic elements provide new experimental opportunities in Non – Hermitian Photonics!

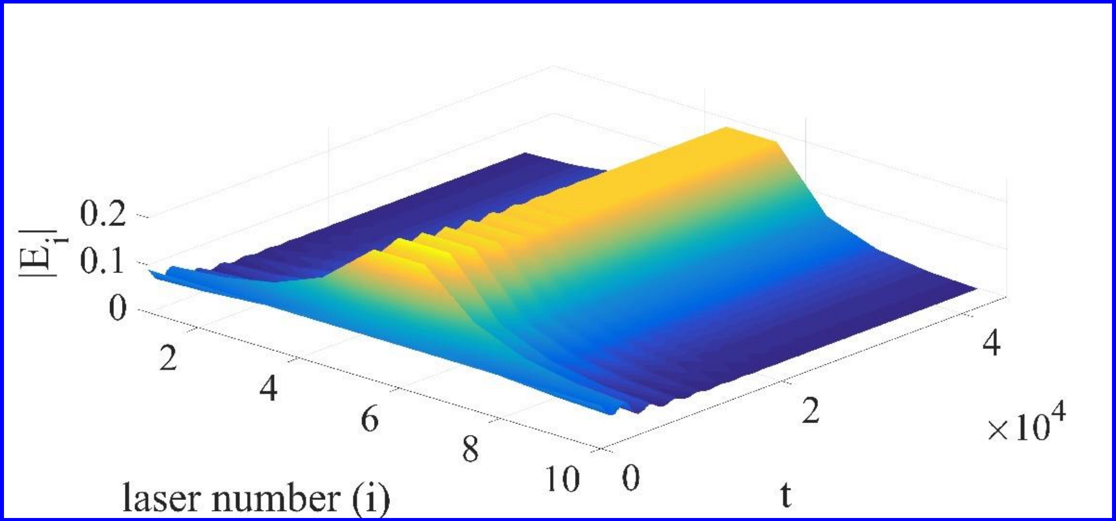
- What new discoveries now await us?

Future Outlook

- What are the properties of wave propagation in asymmetric photonic structures?

- What about connecting many such photonic elements in a lattice?

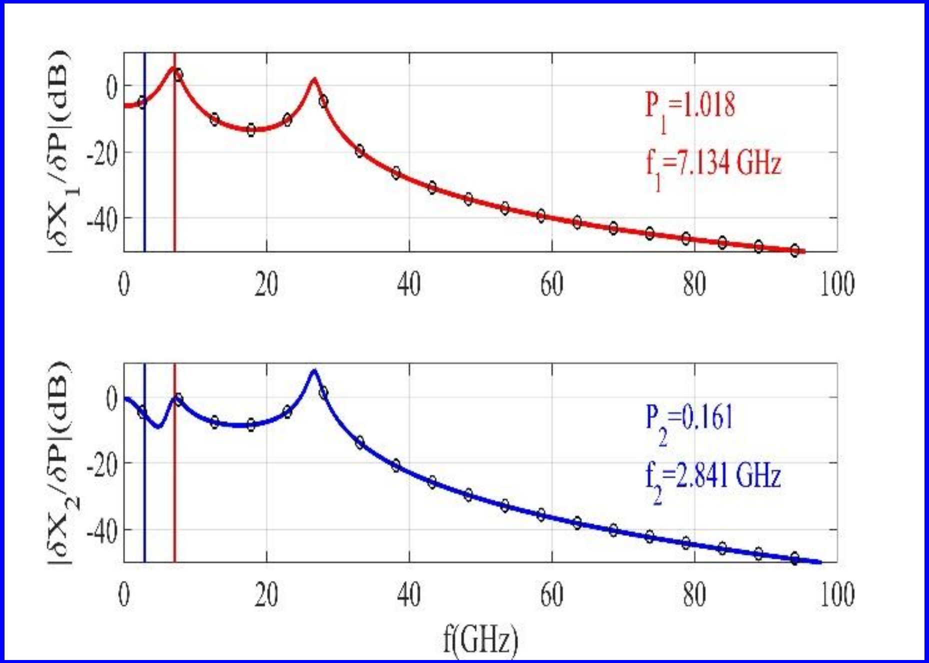
- What if we connect many Couplers of Optical Waveguides and excite 2-3 of the central ones? Stable localized states are found!



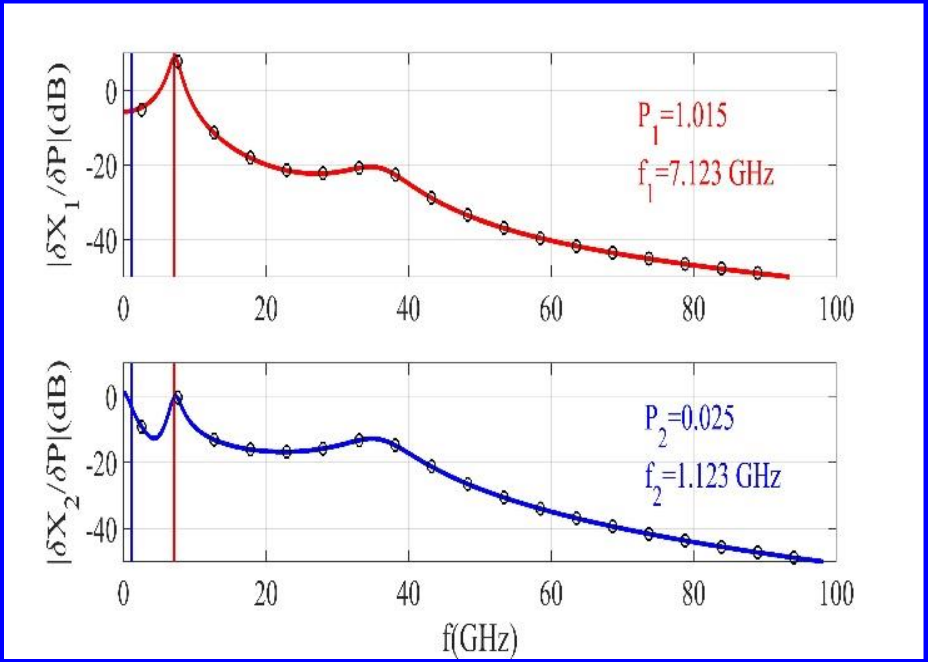
PERIODIC MODULATION OF PUMPING RATES

$$P_i = P^{(0)}_i + (-1)^{i+1}(\delta P)\sin\omega t \qquad i=1,2$$

And if we increase asymmetry between the lasers, we find a big increase of the system's relaxation oscillation frequency which, for increasing asymmetry can reach 40 and more GHz!



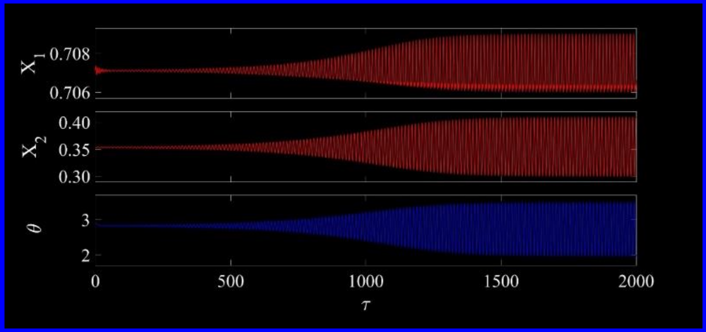
ρ=0.45



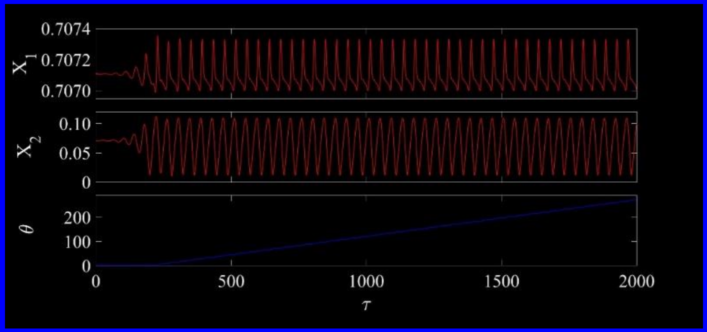
ρ=0.30

Coupling Stable Limit Cycles

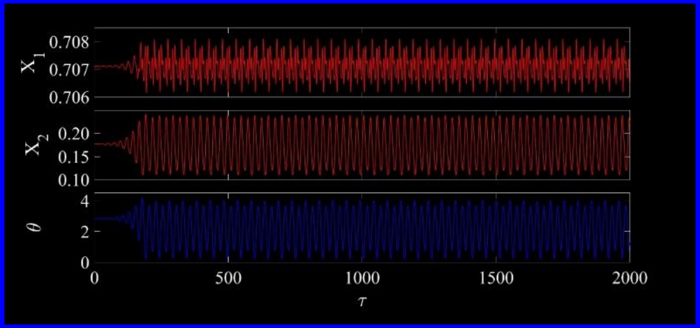
(a)



(b)



(c)



Και δεν θα ήταν ενδιαφέρον, αν βάζαμε πολλούς λειζερς συζευγμένους σε ένα δίκτυο, να εκτελούν μια από τις παραπάνω (a), (b), (c) ταλαντώσεις, να δούμε αν αυξάνοντας τη σύζευξη μεταξύ τους θα μπορούσαμε να έχουμε συγχρονισμό ή chimera states?

Asymmetry reminds us of the poem
"The Road not Taken" by Robert Frost:

I will be saying this with a sigh,
Ages and ages hence,
Two roads diverged in the woods, and I,
I took the one less travelled by,
And that has made all the difference!

References

1. Y. Kominis, T. Bountis and S. Flach, “The Asymmetric Active Coupler: Nonlinear Supermodes and Directed Transport”, Scientific Reports 2016; 6: 33699.
2. Y. Kominis, T. Bountis and S. Flach, “Stability Through Asymmetry: Modulationally Stable Nonlinear Supermodes of Asymmetric non-Hermitian Structures”, Phys. Rev. A 95, 063832 (2017).
3. Y. Kominis, V. Kovanis and T. Bountis, “Controllable Asymmetric Phase-Locked States in Coupled Semiconductoror Lasers”, Phys. Rev. A, 96, 043836 (2017).
4. Y. Kominis, V. Kovanis and T. Bountis, “Spectral Signatures of Exceptional Points and Bifurcations in the Fundamental Active Photonic Dimer”, Phys. Rev. A 96, 053837 (2017).
5. F. Parastesh, S. Jafari, H. Azarnoush, B. Hatef, A. Bountis, “Imperfect Chimera in a Ring of 4 – D Simplified Lorenz Systems”, Chaos, Solitons and Fractals 110 (2018) 203–208 (2018).