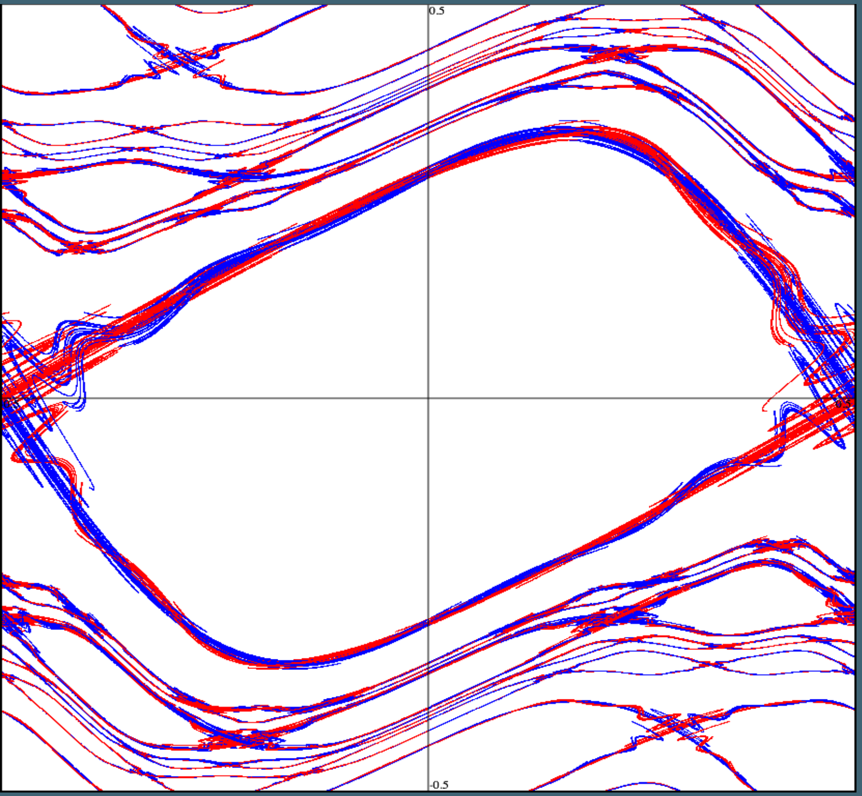
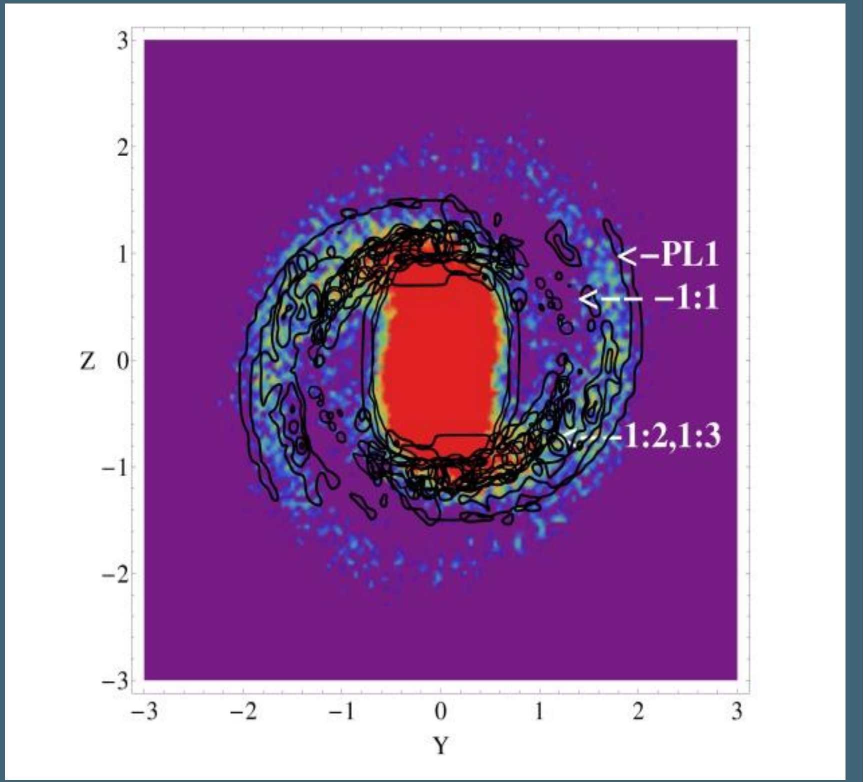
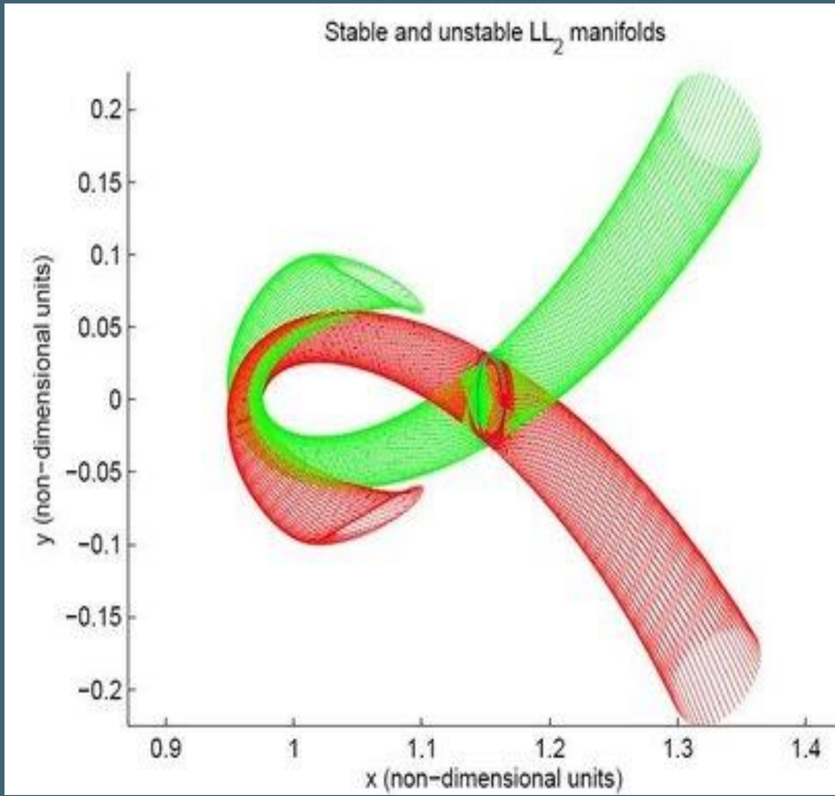
The role of chaos in barred spiral galaxies







Mirella Harsoula
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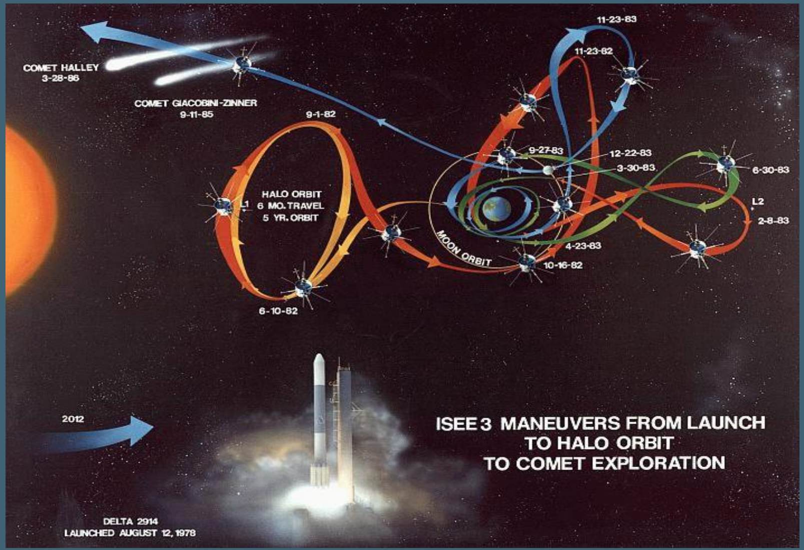
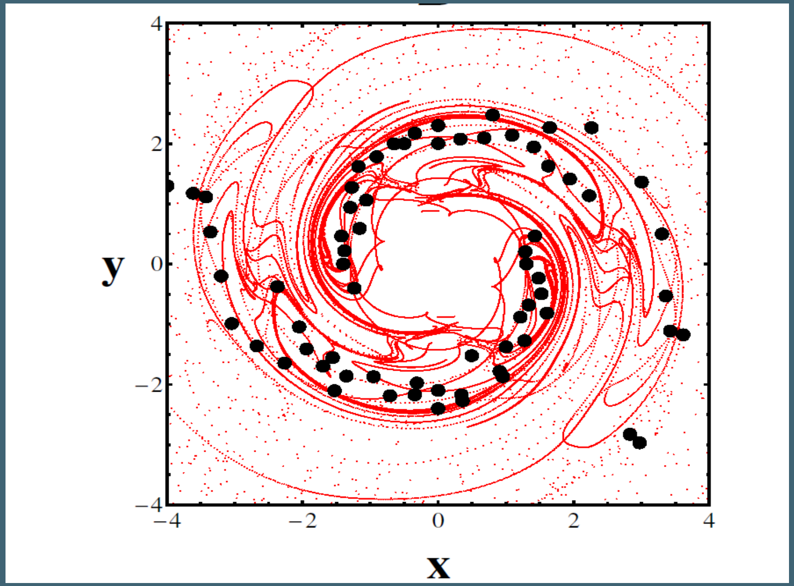
Stable and unstable manifolds

- *Celestial Mechanics:*
- Important in space missions: natural channels of motions for spacecrafts that can save fuel! ("Space Manifold Dynamics" Ferraz-Mello 2010)

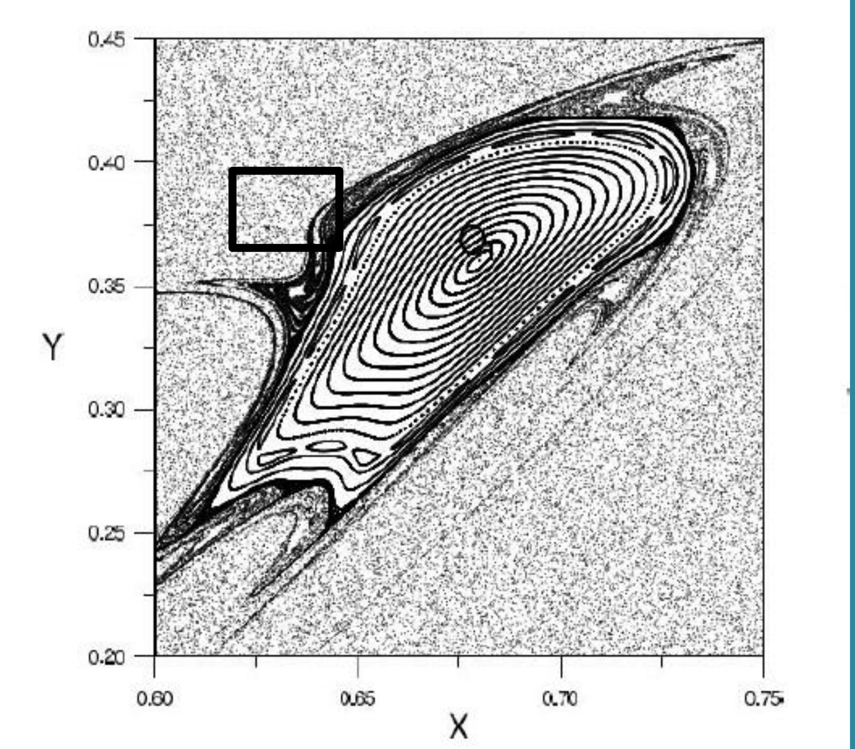
- First mission in 1978 was the International Sun-Earth Explorer 3 (ISEE3), orbited the Sun-Earth L1
- Genesis project was the first to plan an entire mission using manifolds to connect the L1 Halo orbits with the L2 point and Earth for a low-energy round-trip flight.
- *Dynamical Astronomy:*

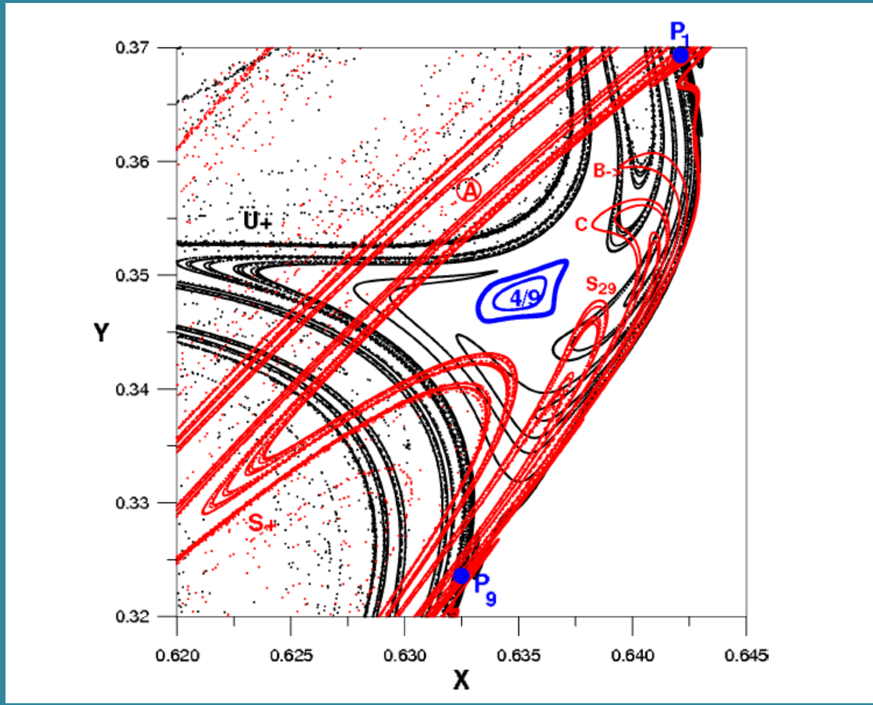
The "*manifold theory*" : chaotic orbits support the spiral structure in barred-spiral galaxies (Voglis et al. 2006, Romero-Gomez et al. 2006)

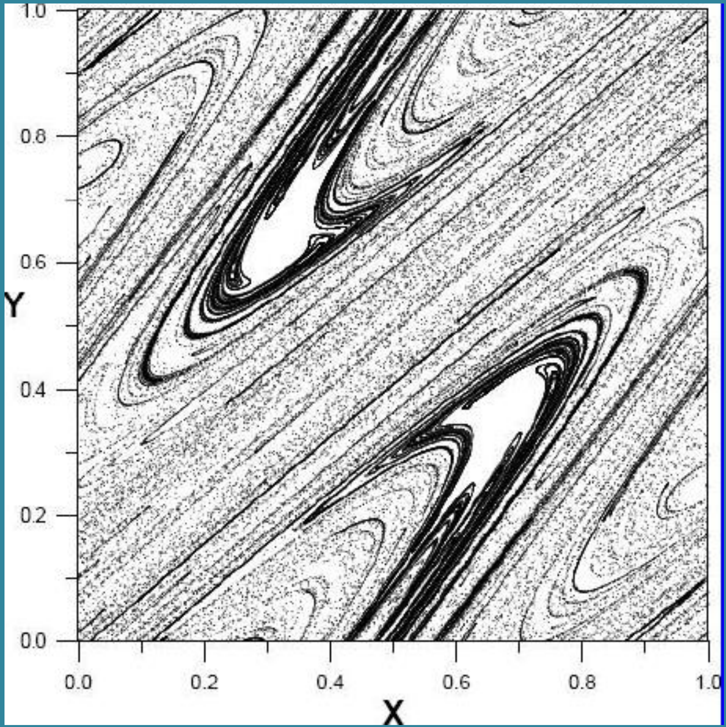




Order and Chaos in Dynamical Systems







$$x' = x + y'$$
$$y' = y + \frac{k}{2\pi}\sin(2\pi x) \qquad (\text{mod } 1)$$

Standard map

STICKINESS IN CHAOS
2008

G. CONTOPOULOS and M. HARSOULA
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Analytical Moser Invariant curves

"The analytic invariants of an Area-Preserving Mapping Near a Hyperbolic Fixed Point Jurgen Moser" (1956)

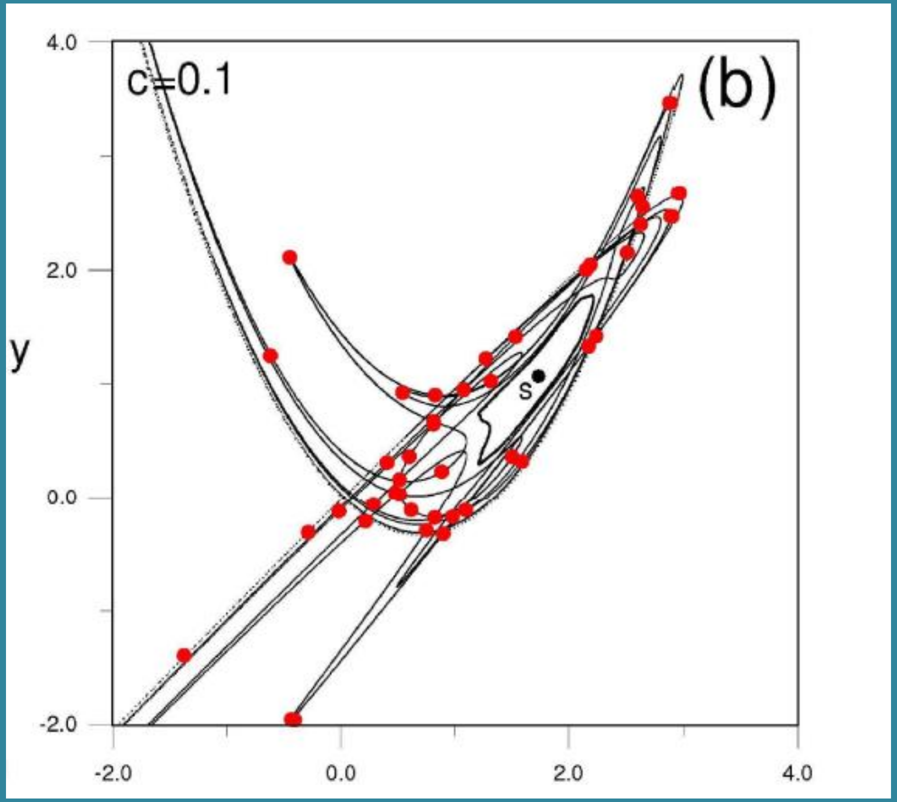
"In this paper we want to determine the analytic invariants of area preserving mappings in the neighborhood of a fixed point"
1958-> Hamiltonians

- Da Silva Ritter et al (1987), Giorgilli (2001), Bongini et al. (2001)

- **The invariant manifolds can be represented by convergent formal series in mappings and in Hamiltonian *models* *C. Efthymiopoulos, G. Contopoulos, and M. Katsanikas, 2014, Celestial Mechanics and Dynamical Astronomy***

- **Analytical description of the structure of chaos. *M. Harsoula G. Contopoulos, and C. Efthymiopoulos, 2015, Journal of Physics A***

- **Convergence regions of the Moser normal forms and the structure of chaos *G. Contopoulos, and M. Harsoula, 2015, Journal of Physics A***



Moser invariant curves in mappings

Hénon symplectic map (Hénon 1969)

$$x' = \cosh(\kappa)x + \sinh(\kappa)y - \frac{\sqrt{2}}{2}\sinh(\kappa)x^2$$

$$y' = \sinh(\kappa)x + \cosh(\kappa)y - \frac{\sqrt{2}}{2}\cosh(\kappa)x^2$$

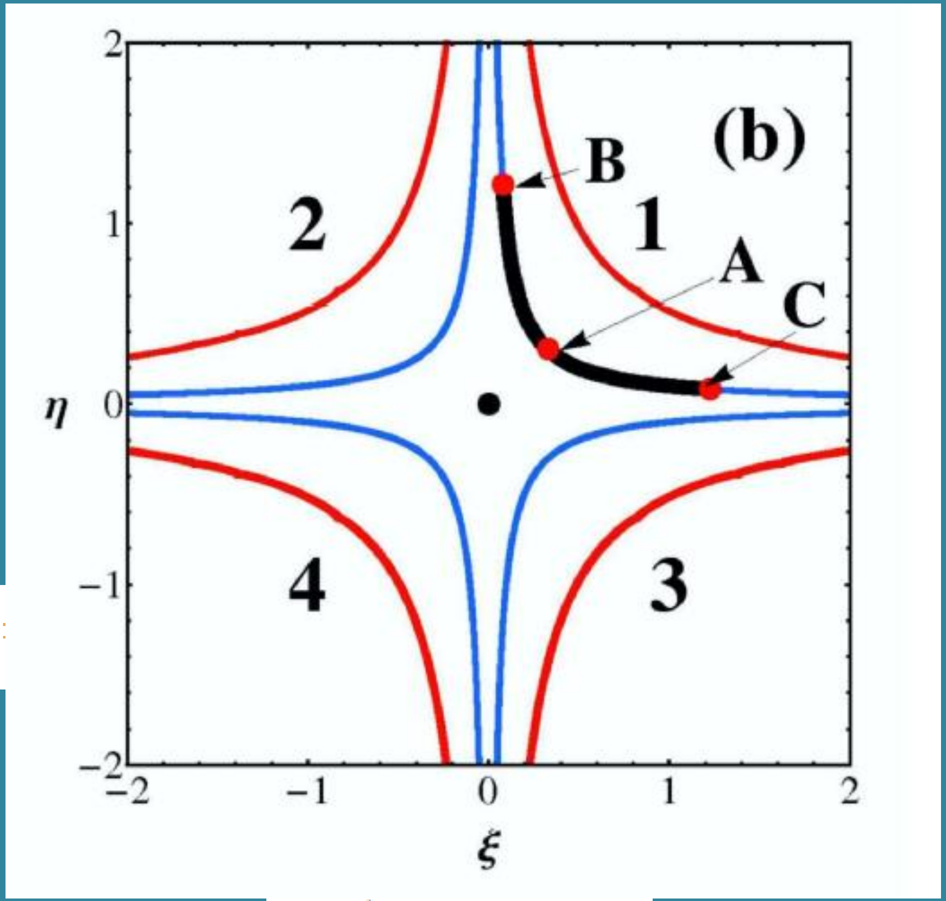
$\Phi = (\Phi_1, \Phi_2)$ of the form $x = (u+v)/\sqrt{2}$, y

$$u = \Phi_1(\xi,\eta) = \xi + \Phi_{1,2}(\xi,\eta) + \dots$$

$$v = \Phi_2(\xi,\eta) = \eta + \Phi_{2,2}(\xi,\eta) + \dots$$

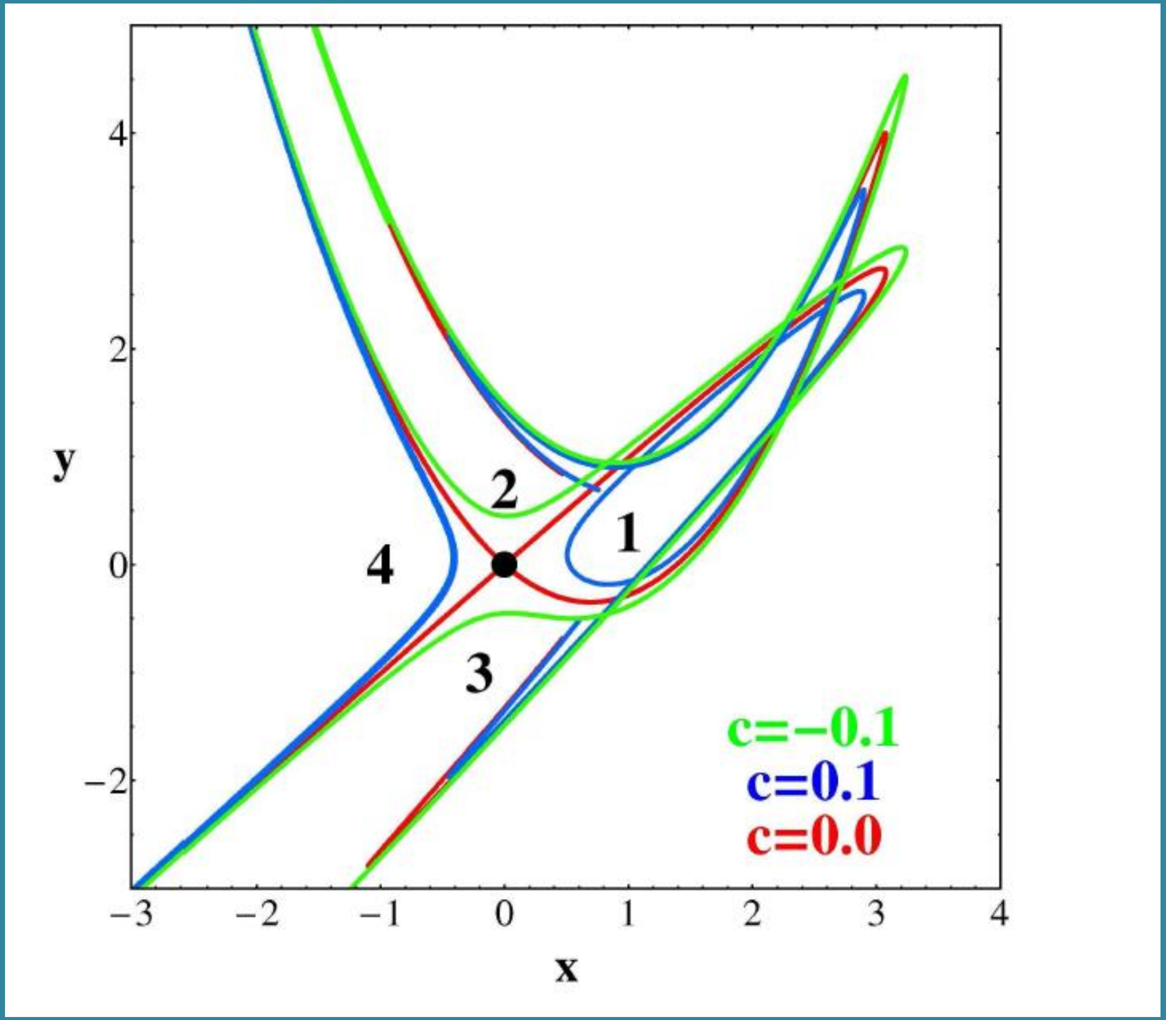
$$\Lambda(c) = \lambda_1 + w_2 c + w_3 c^2 + \dots$$

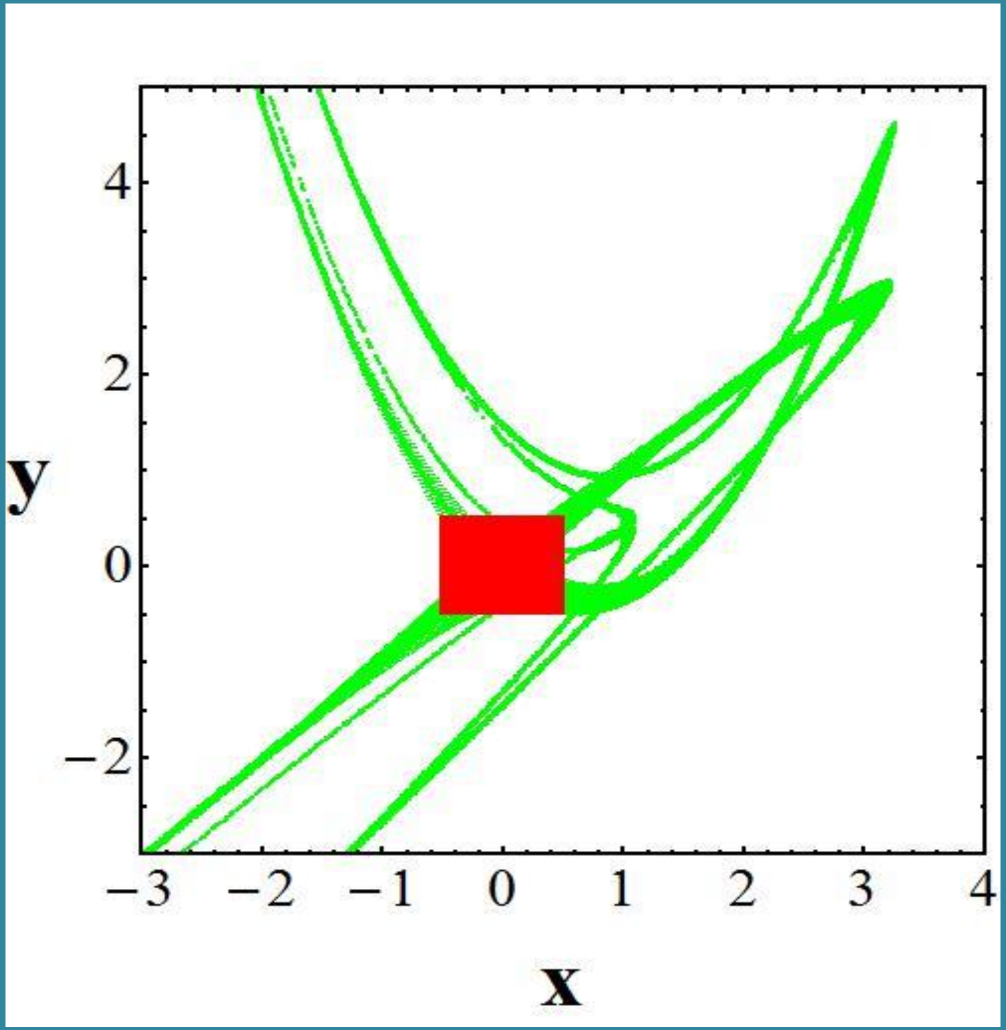
$$\frac{1}{\Lambda(c)} = \lambda_2 + v_2 c + v_3 c^2 + \dots$$



$$\xi' = \Lambda(c)\xi$$

$$\eta' = \frac{1}{\Lambda(c)}\eta$$


y
x
4
2
0
−2
−3
−2
−1
0
1
2
3
4
1
2
3
4
c=−0.1
c=0.1
c=0.0


y
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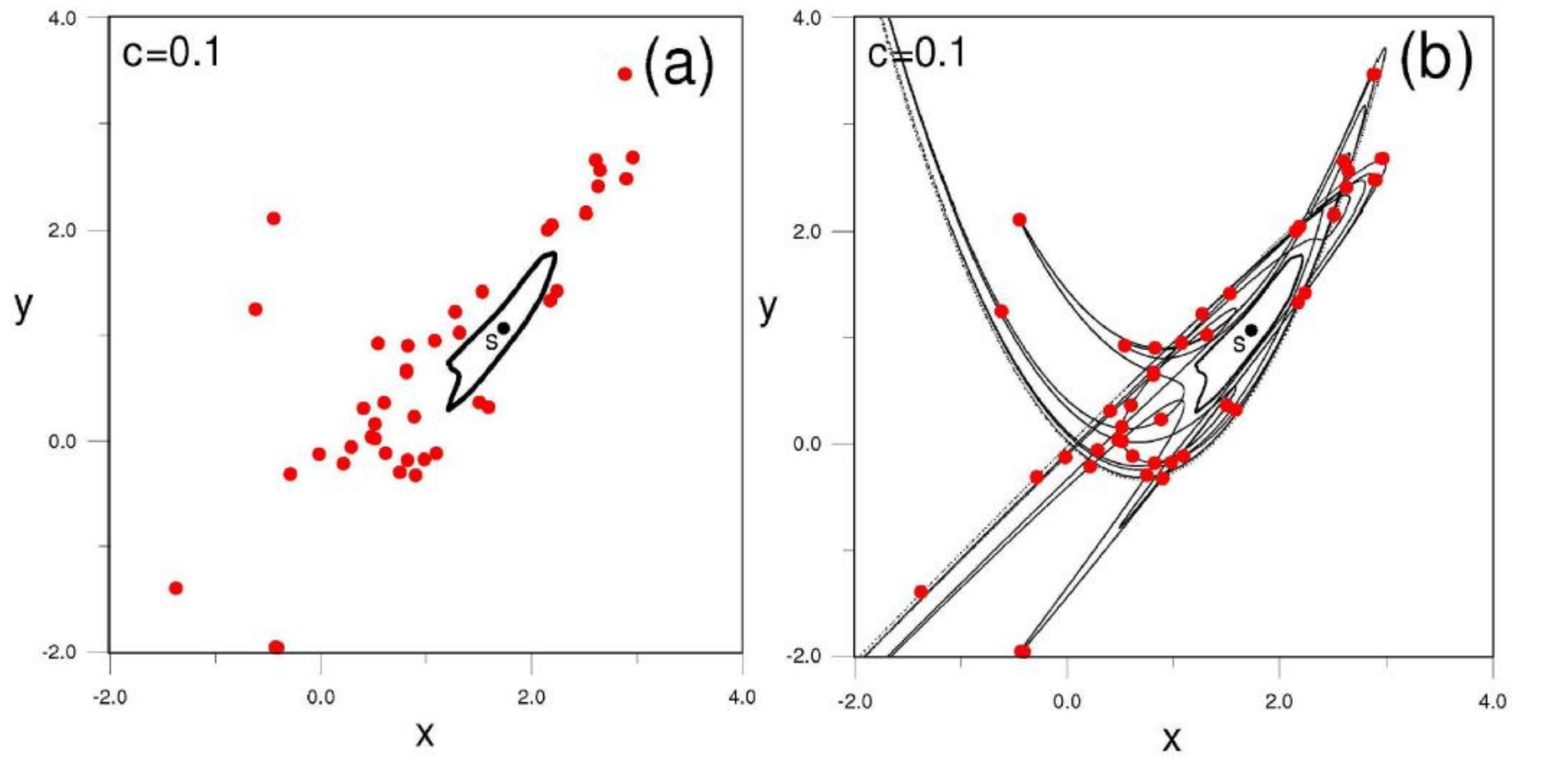
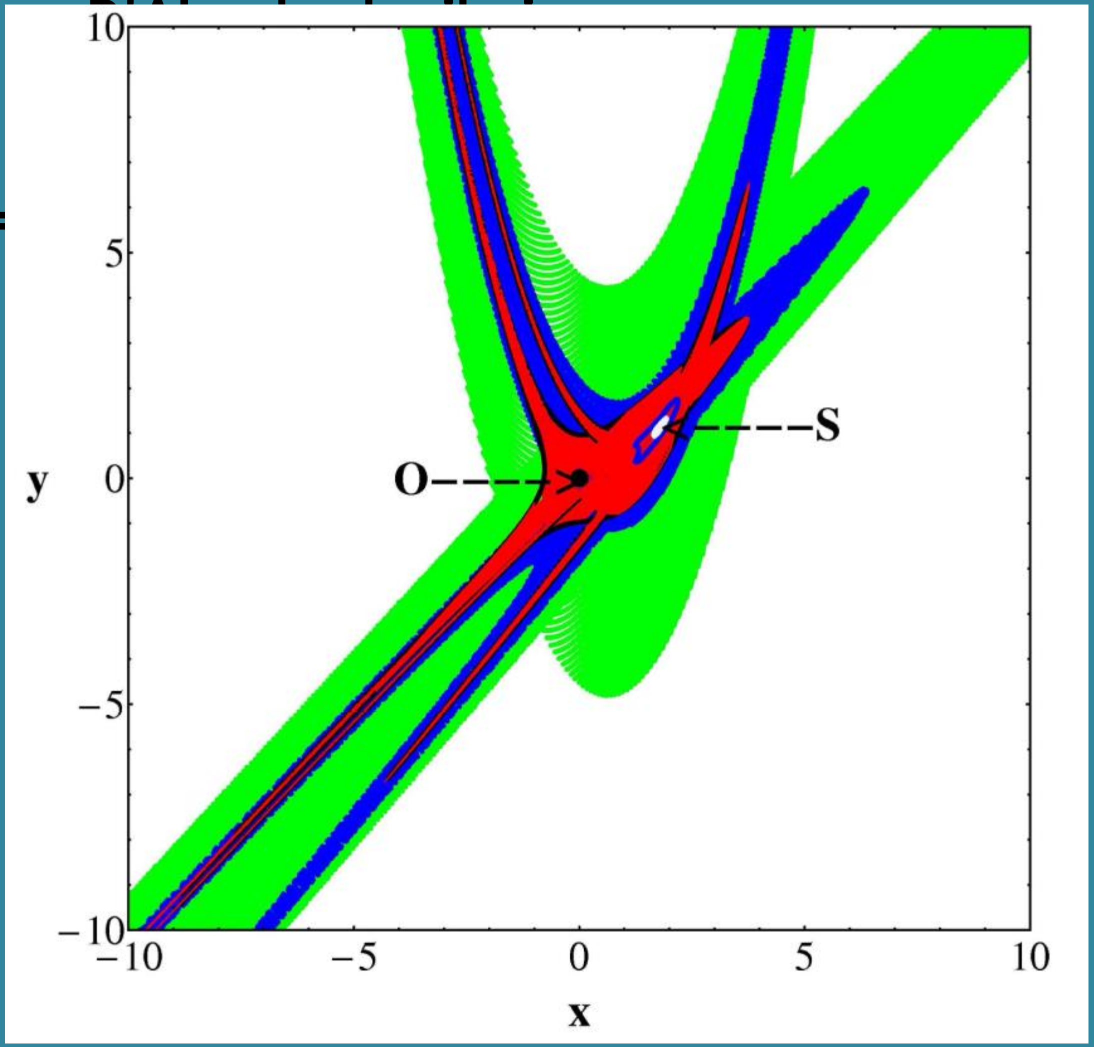
4

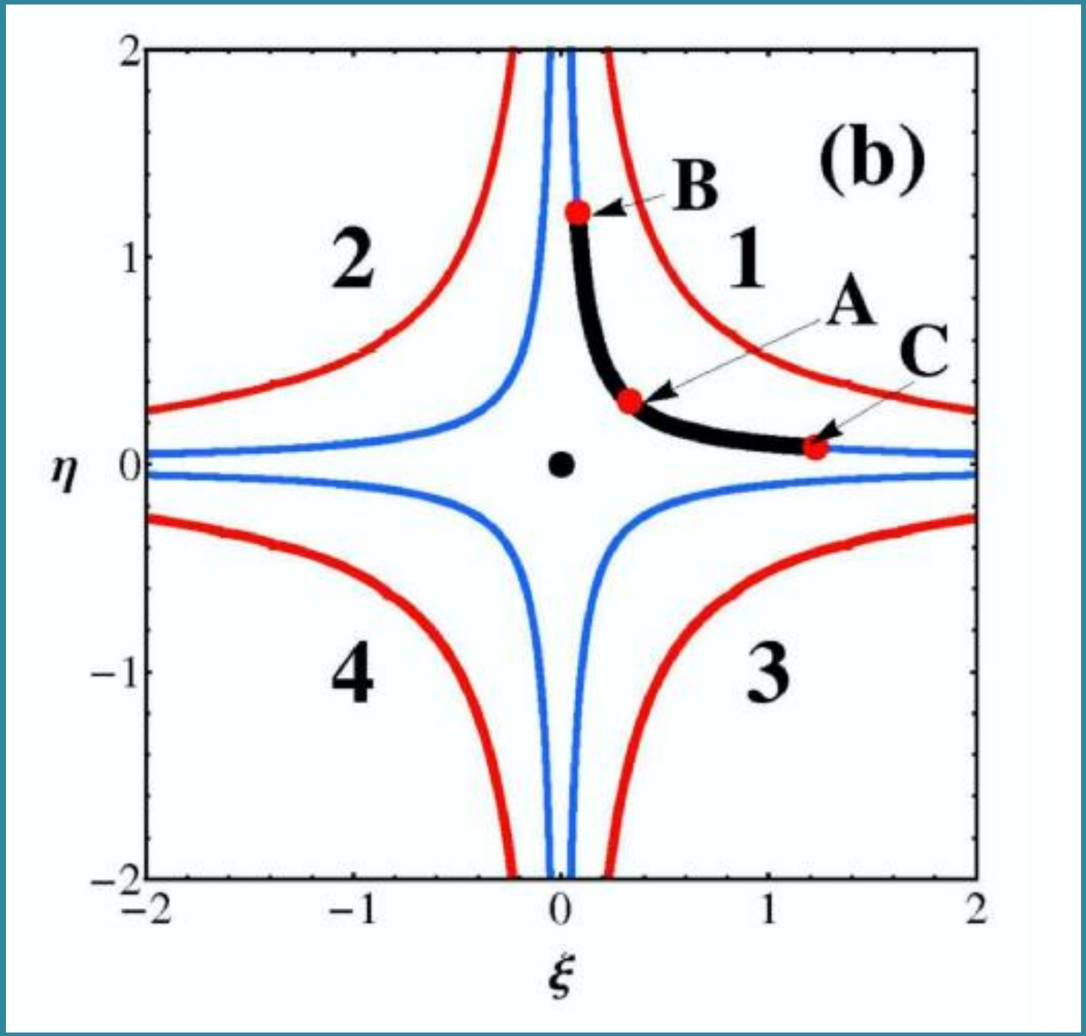
The road of chaos



Moser domain of convergence

$\Phi_1=$





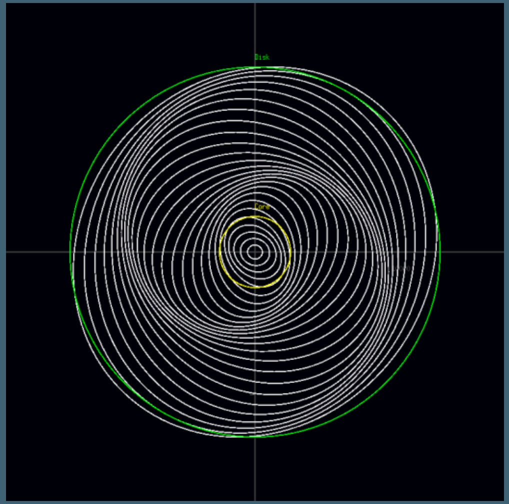
Application in Barred-Spiral galaxies

Normal spiral galaxies

NGC628

NGC5247

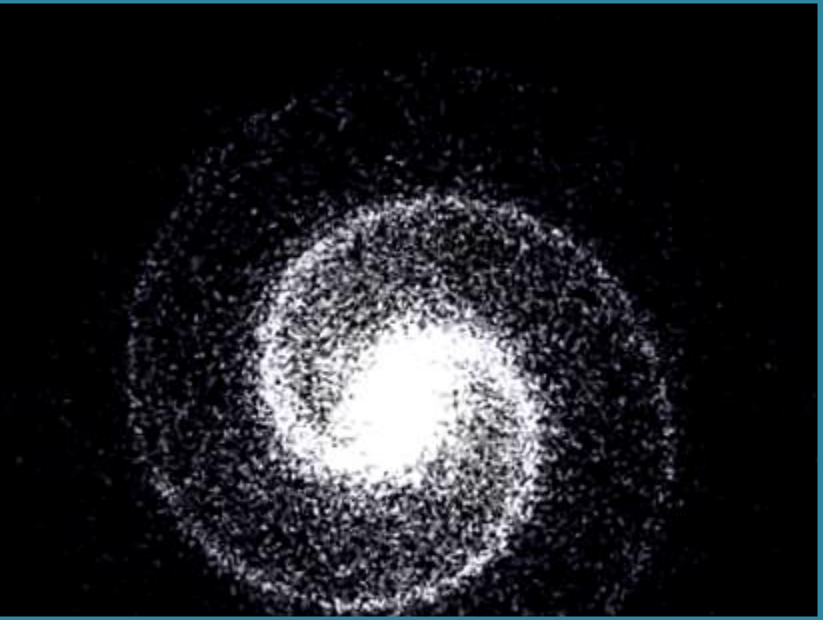
Density waves



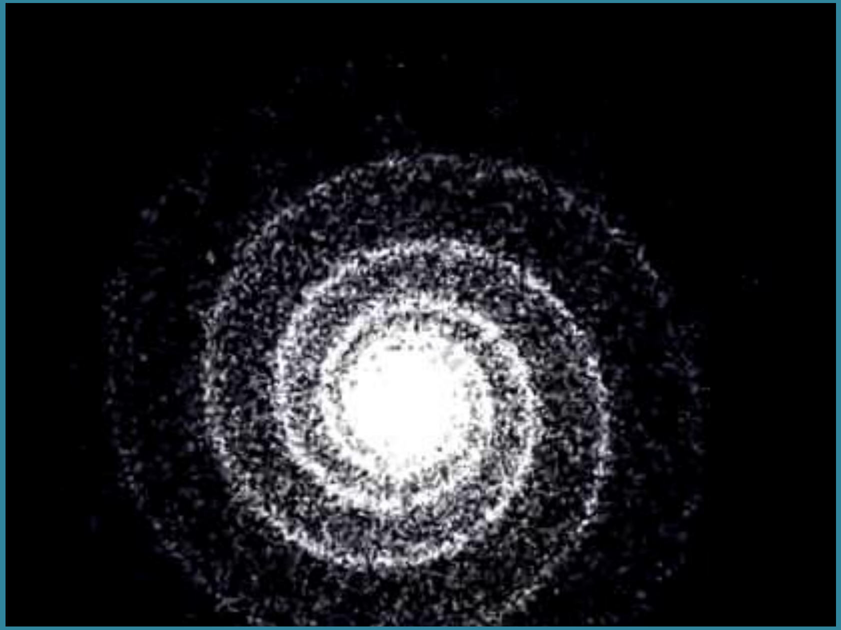
Lindblad 1956 και Lin and Shu 1964

Ποιος μηχανισμός δημιουργεί τις σπείρες;

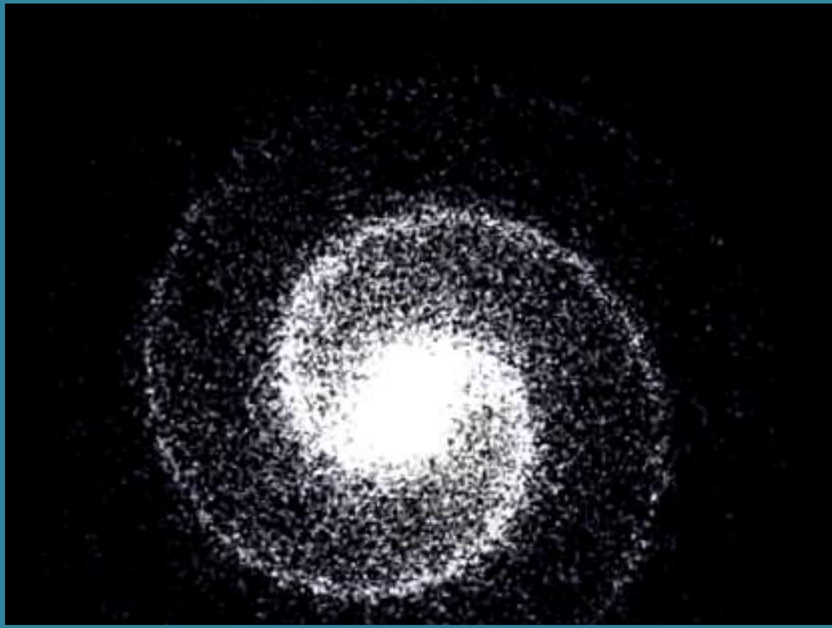
Στέρεο σώμα



Διαφορική περιστροφή

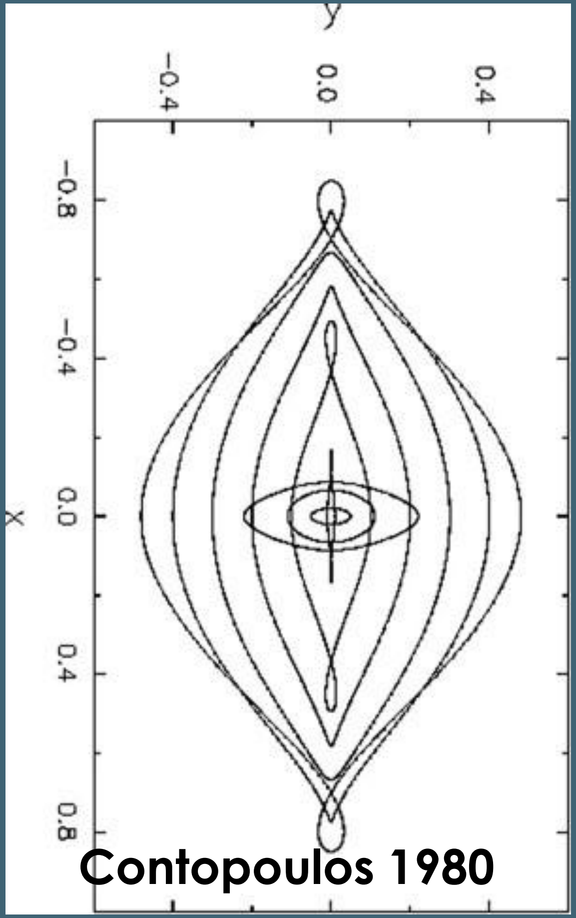


Κύμα πυκνότητας



Lindblad 1956 και Lin and Shu 1964

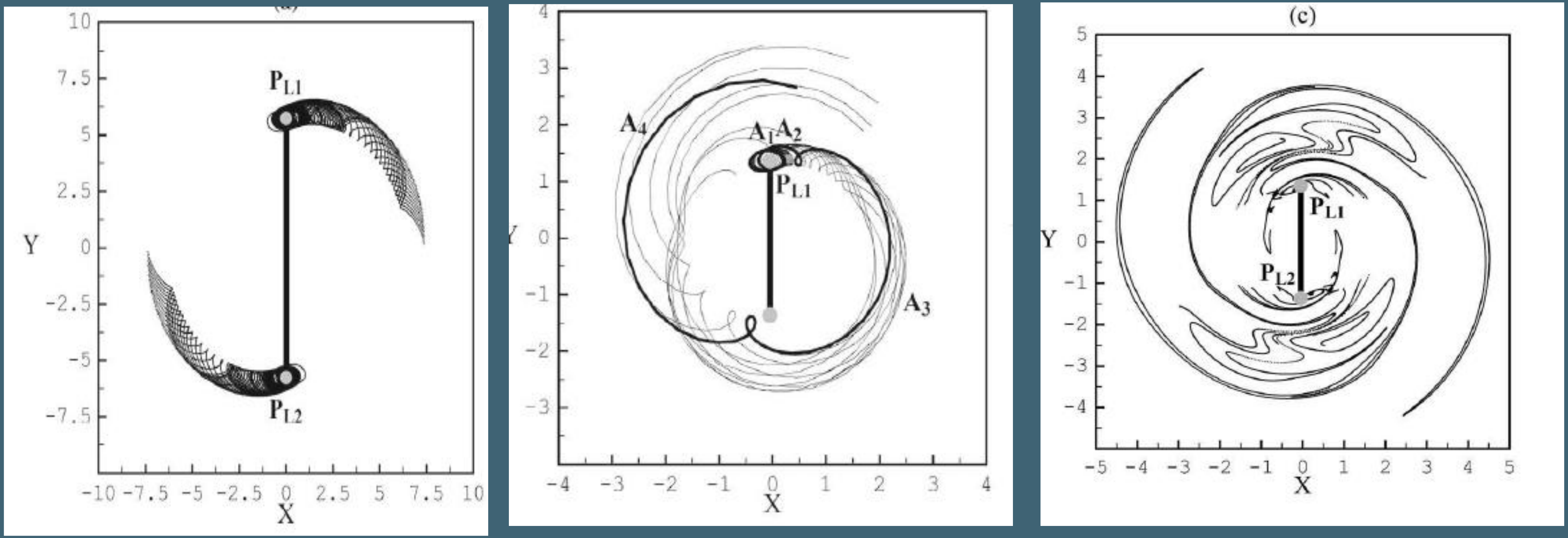
Application in Barred-Spiral galaxies



Contopoulos 1980


NGC 1300

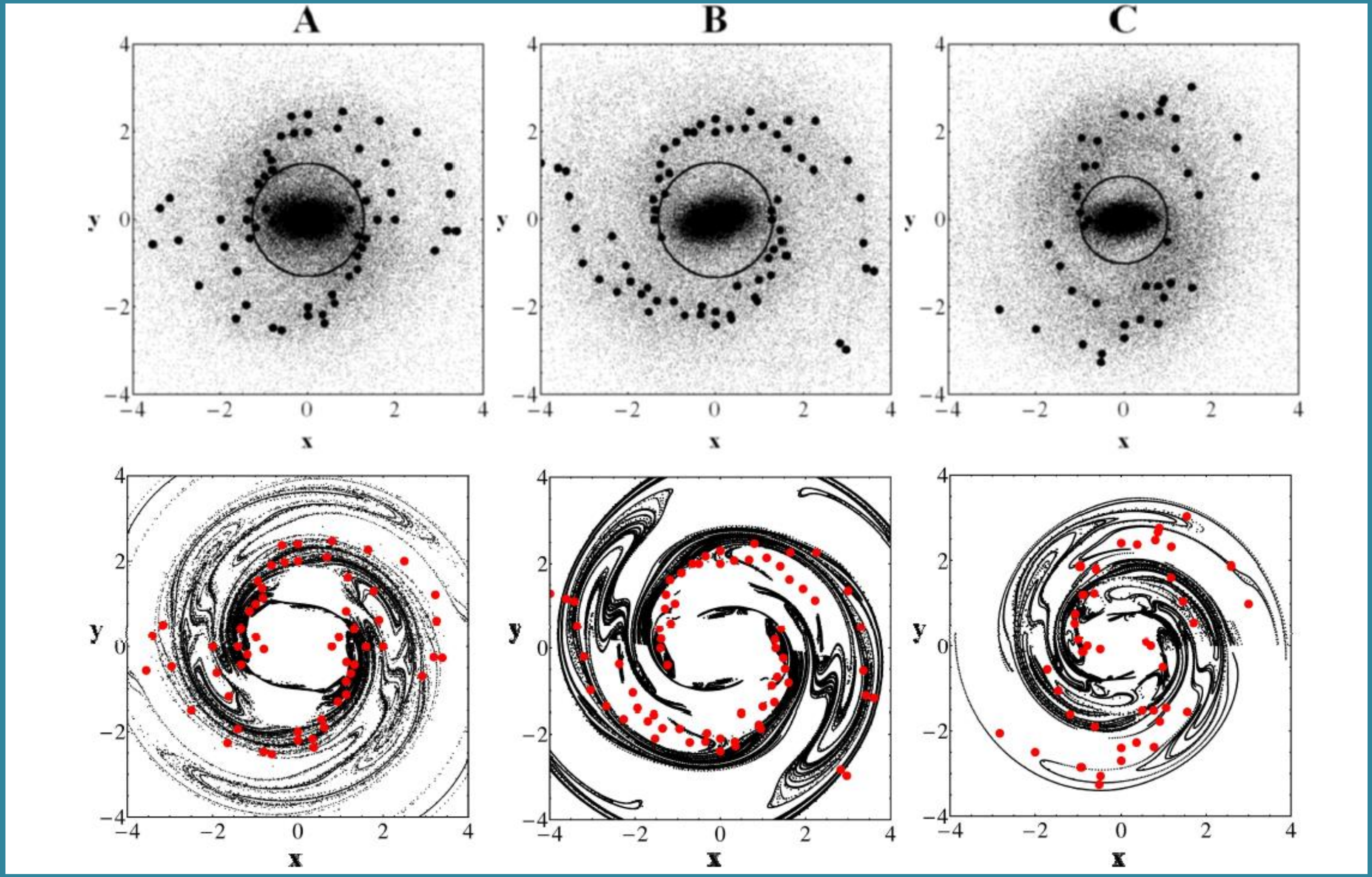
Application in Barred-Spiral galaxies

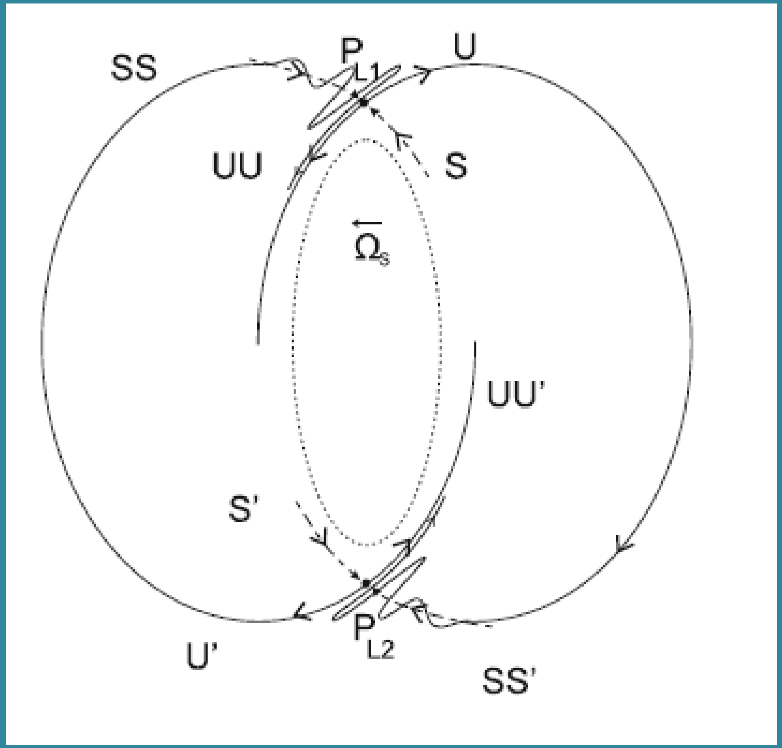


PL1 and PL2 orbits and asymptotic orbits on the unstable manifold in a barred-spiral galactic model

(**Tsoutsis, et al. 2009**)

The manifold theory for spiral arms





Harsoula, Efthymiopoulos and Contopoulos, MNRAS, 2016

The manifold theory and the Moser domain of convergence

$$H = \frac{1}{2}\left(p_x^2 + p_y^2\right) - \Omega_p(xp_y - yp_x) + \Phi(x, y)$$

Lagrangian points: $x_{L1}, y_{L1}, p_{x_{L1}}, p_{y_{L1}}$:

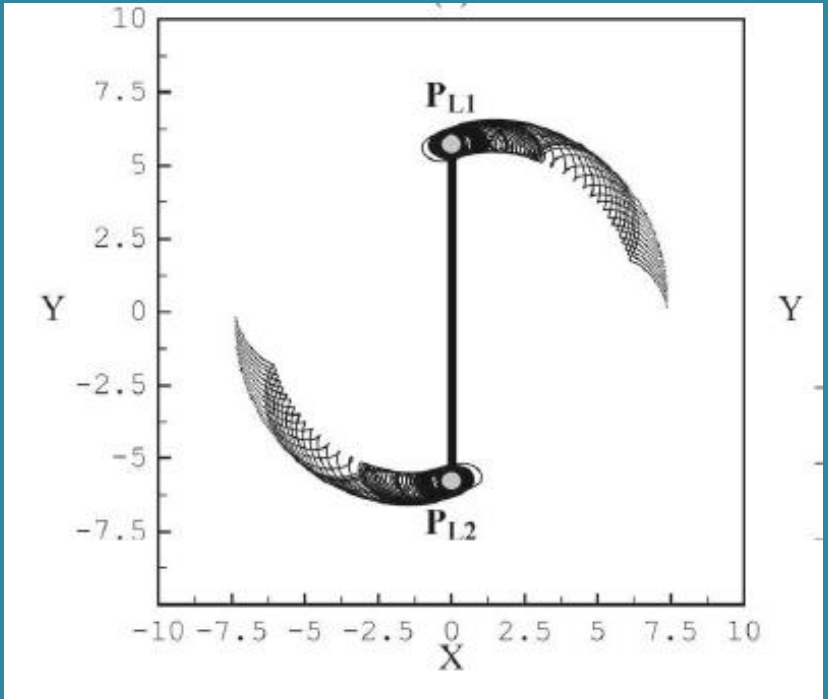
$$\frac{dx}{dt} = \frac{dy}{dt} = \frac{dp_x}{dt} = \frac{dp_y}{dt} = 0$$

$$\lambda_{1,2} = \pm\,\iota\,\omega_0 \qquad \lambda_{3,4} = \pm\,\nu_0$$

$$(x, y, p_x, p_y) \rightarrow (q, u, p, v)$$

$$H = \omega_0\left(\frac{q^2 + p^2}{2}\right) + \nu_0 uv + \sum_{s=3}^{\infty} P_s(q, p, u, v)$$

$$u = u_0 e^{\nu_0 t} \qquad v = v_0 e^{-\nu_0 t}$$



“Moser” normal form construction

$$H = \frac{P_r^2}{2} + \frac{P_\phi^2}{2r^2} - \Omega_p \; P_\phi + \Phi(r,\phi)$$

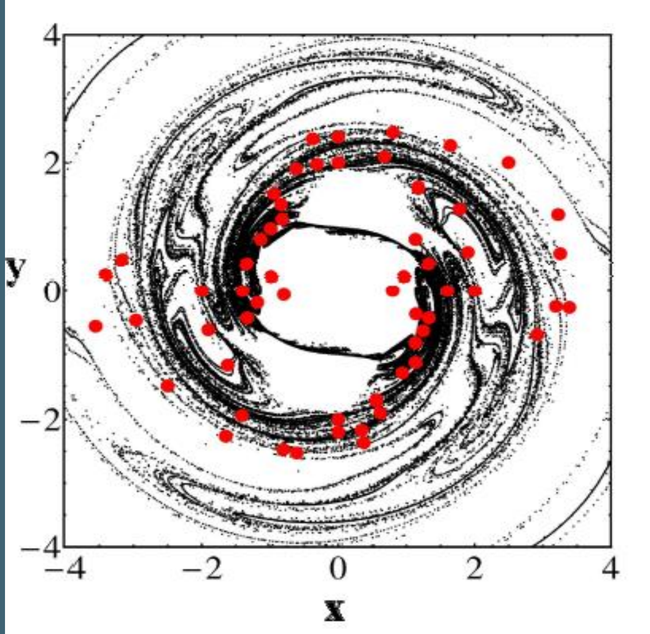
$$\Phi(r,\phi) = \Phi_0(r) + \Phi_1(r)\cos 2\phi + \Phi_2(r)\sin 2\phi$$

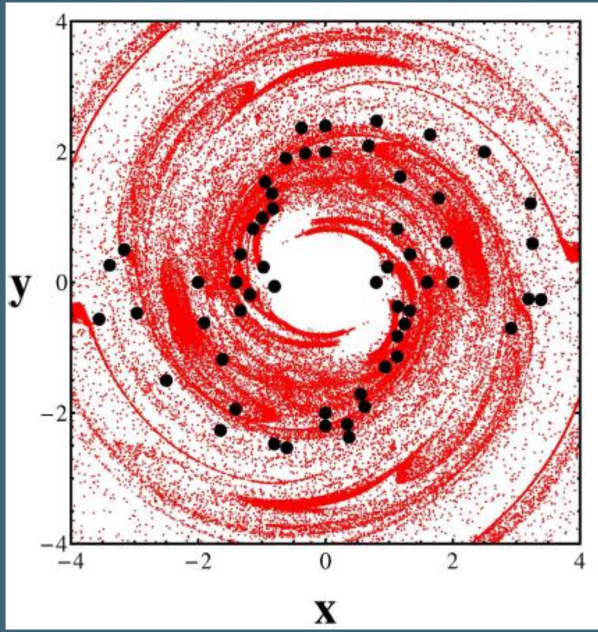
$$r \to r_{L_1} + \delta r, \;\; P_r \to P_{r_{L_1}} + P_x, \;\; \phi \to \phi_{L_1} + \delta\phi, \;\; P_\phi \to P_{\phi_{L_1}} + J_\phi$$

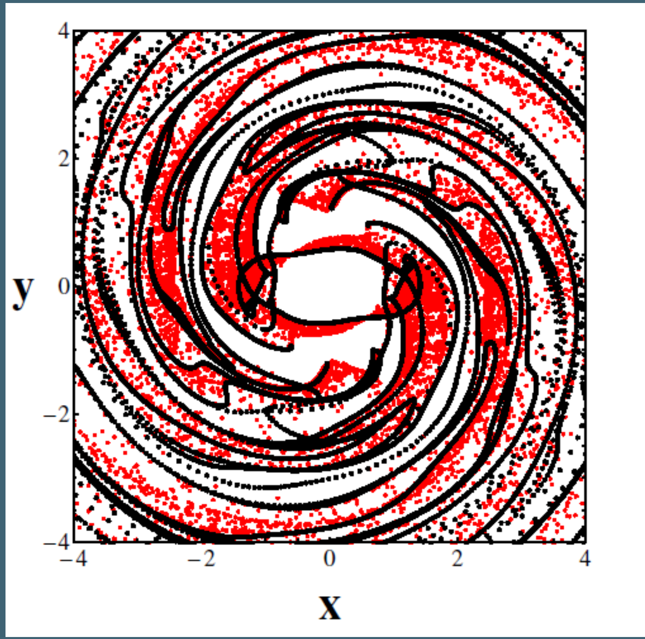
$$\begin{pmatrix} \dot{\delta r} \\ \dot{\delta\phi} \\ \dot{P}_x \\ \dot{J}_\phi \end{pmatrix} = M \begin{pmatrix} \delta r \\ \delta\phi \\ P_x \\ J_\phi \end{pmatrix}$$

$$\begin{aligned} Z(I = iab, c = \xi\eta) = i\omega_0 ab + \nu_0\xi\eta + \zeta_{21}a^2b^2 + \zeta_{22}\xi^2\eta^2 \\ +\zeta_{23}ab\xi\eta + \zeta_{31}a^3b^3 + \zeta_{32}ab\xi^2\eta^2 \\ +\zeta_{33}q^2p^2\xi\eta + \zeta_{34}\xi^3\eta^3 + \ldots \end{aligned}$$

The extended Moser domains of convergence







Thank you