The role of chaos in barred spiral galaxies

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Stable and unstable manifolds

- **Celestial Mechanics:**
  - Important in space missions: natural channels of motions for spacecrafts that can save fuel! (“Space Manifold Dynamics” Ferraz-Mello 2010)

- First mission in 1978 was the International Sun-Earth Explorer 3 (ISEE3), orbited the Sun-Earth L1

- Genesis project was the first to plan an entire mission using manifolds to connect the L1 Halo orbits with the L2 point and Earth for a low-energy round-trip flight.

- **Dynamical Astronomy:**
  The “manifold theory” : chaotic orbits support the spiral structure in barred-spiral galaxies (Voglis et al. 2006, Romero-Gomez et al. 2006)
Order and Chaos in Dynamical Systems

Standard map

\[ x' = x + y' \quad (\text{mod } 1) \]

\[ y' = y + \frac{k}{2\pi} \sin(2\pi x) \]
Analytical Moser Invariant curves

“The analytic invariants of an Area-Preserving Mapping Near a Hyperbolic Fixed Point Jurgen Moser” (1956)

“In this paper we want to determine the analytic invariants of area preserving mappings in the neighborhood of a fixed point”

1958-> Hamiltonians


• The invariant manifolds can be represented by convergent formal series in mappings and in Hamiltonian models. C. Efthymiopoulos, G. Contopoulos, and M. Katsanikas, 2014, Celestial Mechanics and Dynamical Astronomy


• Convergence regions of the Moser normal forms and the structure of chaos G. Contopoulos, and M. Harsoula, 2015, Journal of Physics A
Moser invariant curves in mappings

Hénon symplectic map (Hénon 1969)

\[ x' = \cosh(\kappa)x + \sinh(\kappa)y - \frac{\sqrt{2}}{2} \sinh(\kappa)x^2 \]
\[ y' = \sinh(\kappa)x + \cosh(\kappa)y - \frac{\sqrt{2}}{2} \cosh(\kappa)x^2 \]

\[ \Phi = (\Phi_1, \Phi_2) \text{ of the form } x = (u + v)/\sqrt{2}, \ y: \]

\[ u = \Phi_1(\xi, \eta) = \xi + \Phi_{1,2}(\xi, \eta) + \ldots \]
\[ v = \Phi_2(\xi, \eta) = \eta + \Phi_{2,2}(\xi, \eta) + \ldots \]

\[ \Lambda(c) = \lambda_1 + w_2c + w_3c^2 + \ldots \]
\[ \frac{1}{\Lambda(c)} = \lambda_2 + v_2c + v_3c^2 + \ldots \]

\[ \xi' = \Lambda(c)\xi \]
\[ \eta' = \frac{1}{\Lambda(c)}\eta \]
The road of chaos
Moser domain of convergence

\[ \phi_1 = \text{function} \]

\[ \xi = r \cos(\phi), \quad \eta = r \sin(\phi) \]

Radius of convergence:
Application in Barred-Spiral galaxies

Normal spiral galaxies

NGC628  NGC5247

Density waves

Lindblad 1956 και Lin and Shu 1964
Ποιος μηχανισμός δημιουργεί τις σπείρες;

Στέρεο σώμα

Διαφορική περιστροφή

Κύμα πυκνότητας

Lindblad 1956 και Lin and Shu 1964
Application in Barred-Spiral galaxies
Application in Barred-Spiral galaxies

PL1 and PL2 orbits and asymptotic orbits on the unstable manifold in a barred-spiral galactic model

(Tsoutsis, et al. 2009)
The manifold theory for spiral arms

The manifold theory and the Moser domain of convergence

\[ H = \frac{1}{2} (p_x^2 + p_y^2) - \Omega_p xp_y - yp_x + \Phi(x, y) \]

**Lagrangian points:** \( x_{L1}, y_{L1}, p_{x_{L1}}, p_{y_{L1}} \),

\[ \frac{dx}{dt} = \frac{dy}{dt} = \frac{dp_x}{dt} = \frac{dp_y}{dt} = 0 \]

\[ \lambda_{1,2} = \pm i \omega_0 \quad \lambda_{3,4} = \pm \nu_0 \]

\((x, y, p_x, p_y) \rightarrow (q, u, p, v)\)

\[ H = \omega_0 \left( \frac{q^2 + p^2}{2} \right) + \nu_0 uv + \sum_{s=3}^{\infty} P_s(q, p, u, v) \]

\[ u = u_0 e^{\nu_0 t} \quad v = v_0 e^{-\nu_0 t} \]
“Moser” normal form construction

\[ H = \frac{P_r^2}{2} + \frac{P_\phi^2}{2r^2} - \Omega \cdot P_\phi + \Phi(r, \phi) \]

\[ \Phi(r, \phi) = \Phi_0(r) + \Phi_1(r) \cos 2\phi + \Phi_2(r) \sin 2\phi \]

\[ r \to r_{L_1} + \delta r, \quad P_r \to P_{r_{L_1}} + P_x, \quad \phi \to \phi_{L_1} + \delta \phi, \quad P_\phi \to P_{\phi_{L_1}} + J_\phi \]

\[
\begin{pmatrix}
\dot{\delta r} \\
\dot{\delta \phi} \\
\dot{P_x} \\
\dot{J_\phi}
\end{pmatrix}
= M
\begin{pmatrix}
\delta r \\
\delta \phi \\
P_x \\
J_\phi
\end{pmatrix}
\]

\[ Z(I = iab, c = \xi \eta) = i\omega_0 ab + \nu_0 \xi \eta + \zeta_{21} a^2 b^2 + \zeta_{22} \xi^2 \eta^2 + \zeta_{23} ab \xi \eta + \zeta_{31} a^3 b^3 + \zeta_{32} ab \xi^2 \eta^2 + \zeta_{33} q^2 P^2 \xi \eta + \zeta_{34} \xi^3 \eta^3 + \ldots \]
The extended Moser domains of convergence
Thank you